Magnetic field amplification by a nonlinear electron streaming instability

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Abstract

Relativistic electron beams can generate strong magnetic fields through streaming instabilities such as the Weibel instability. However, when the beam is dilute, the Weibel magnetic fields saturate at low levels.

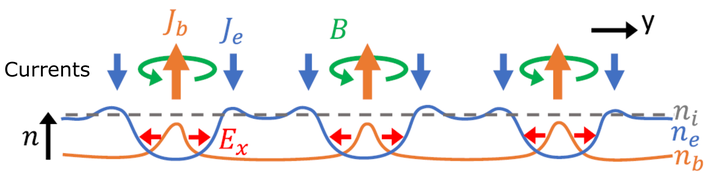
We report a **new nonlinear instability** which can further increase the magnetic field strength and spatial scale by orders of magnitude after saturation of the Weibel instability. We show that this instability may grow in the few-microcoulomb electron beams now available with picosecond kJ-class lasers.

A tale of two instabilities

- Dilute, relativistic e-beam, density n_b, Lorentz factor $\gamma_{b0} \gg 1$
- Dense return electron current, density n_e, Density ratio $\alpha = \frac{n_b}{n_e} \ll 1$
- Initially-stationary ions, density $n_i = n_b + n_e$

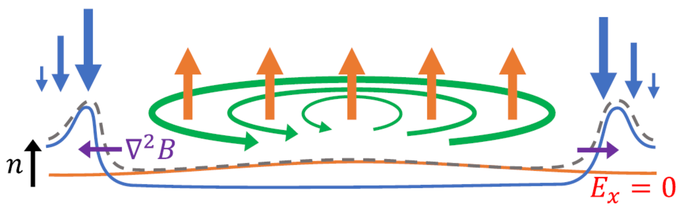
Weibel instability

The Weibel instability is dominant when the beam and background density are comparable but saturates at a low level when the beam is dilute.



Nonlinear instability

A nonlinear streaming instability can arise after Weibel saturates in dilute-beam systems. It grows as the magnetic pressure expels both background species to expose beam current and form large magnetic-field-filled cavities.



In nature

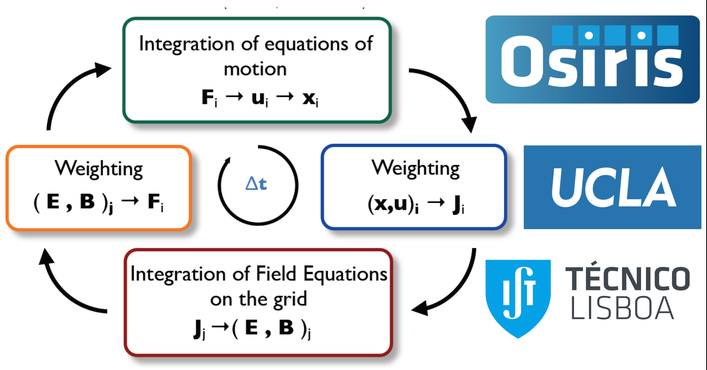
Astrophysical jets

NASA and the Hubble Heritage Team (STScI/AURA)

Gamma-ray bursts

ESO/A. Roquette

Particle-in-cell simulations



Theoretical analysis

Growth rate: $\Gamma = \frac{\alpha}{\sqrt{\delta m_i/m_e}}\omega_p$ ($\delta = 4$ for slab, $\delta = 3$ for cylindrical)

Saturation magnetization (magnetic energy normalized to initial kinetic energy)

$$\epsilon_B \sim \begin{cases} \frac{1}{8}, & \gamma_{b0} < \frac{m_i}{m_e} \\ \frac{m_i}{8\gamma_{b0} m_e}, & \gamma_{b0} > \frac{m_i}{m_e} \end{cases}$$

Saturation magnetic length scale:

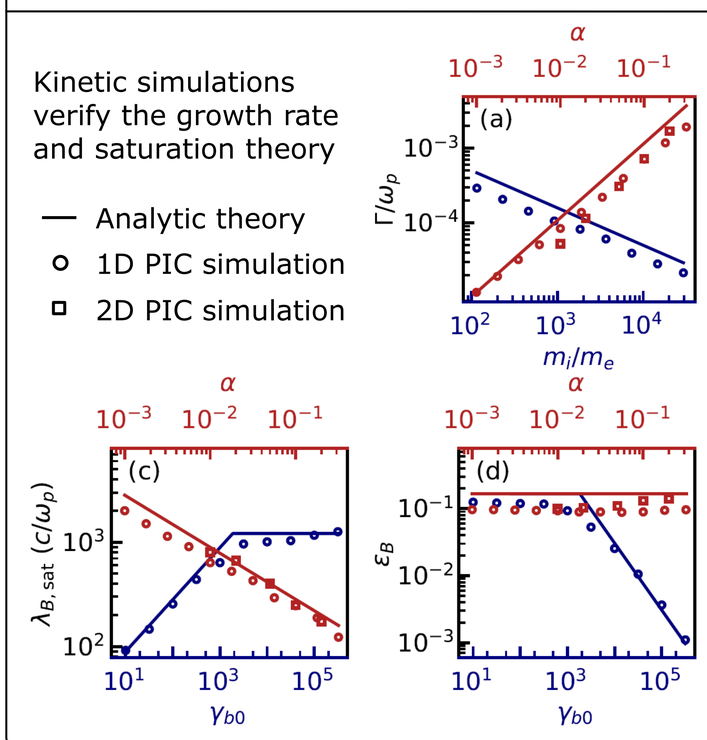
$$\lambda_B \sim \begin{cases} \sqrt{\frac{8\gamma_{b0}}{\alpha}}\frac{c}{\omega_p}, & \gamma_{b0} < \frac{m_i}{m_e} \\ \sqrt{\frac{8 m_i/m_e}{\alpha}}\frac{c}{\omega_p}, & \gamma_{b0} > \frac{m_i}{m_e} \end{cases}$$

With m_i (m_e) the ion (electron) mass and ω_p the background electron plasma frequency.

Theory is validated by 1D, 2D PIC

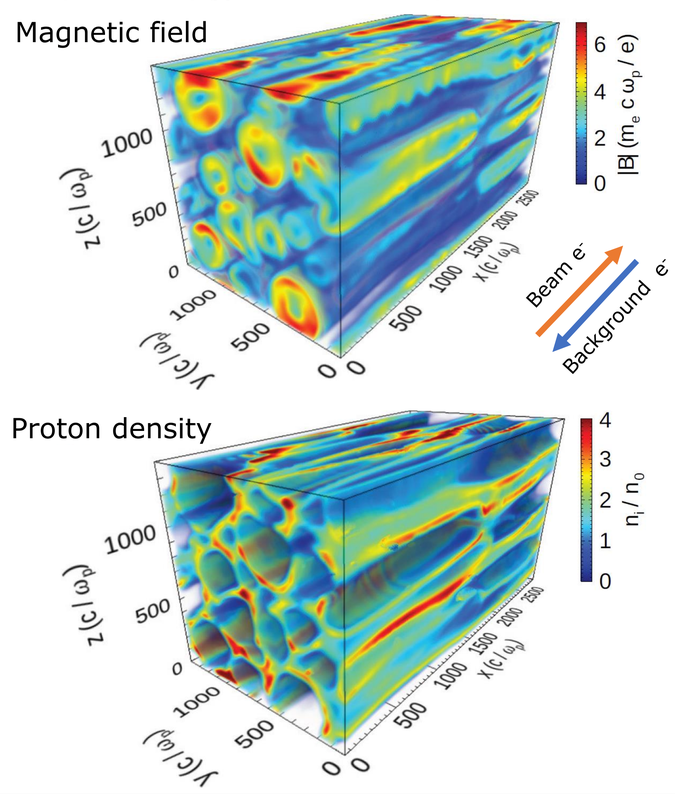
Kinetic simulations verify the growth rate and saturation theory

— Analytic theory
○ 1D PIC simulation
□ 2D PIC simulation



Large B-field coherence length

This 3D simulation demonstrates that the nonlinear instability can generate magnetic fields with length scales 20x larger than Weibel alone can produce ($\gamma_{b0} = 1000, \alpha = 0.1$)



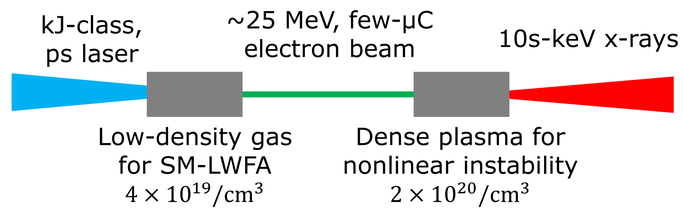
We can drive it with μC e-beams

To observe the nonlinear instability, we need a large beam diameter $D \gg \lambda_B$ and long duration $T \gg 1/\Gamma$. For a 25 MeV beam with density $n_b = 4 \times 10^{19}/\mathrm{cm}^3$ and a background hydrogen density $n_e = 2 \times 10^{20}/\mathrm{cm}^3$, we need

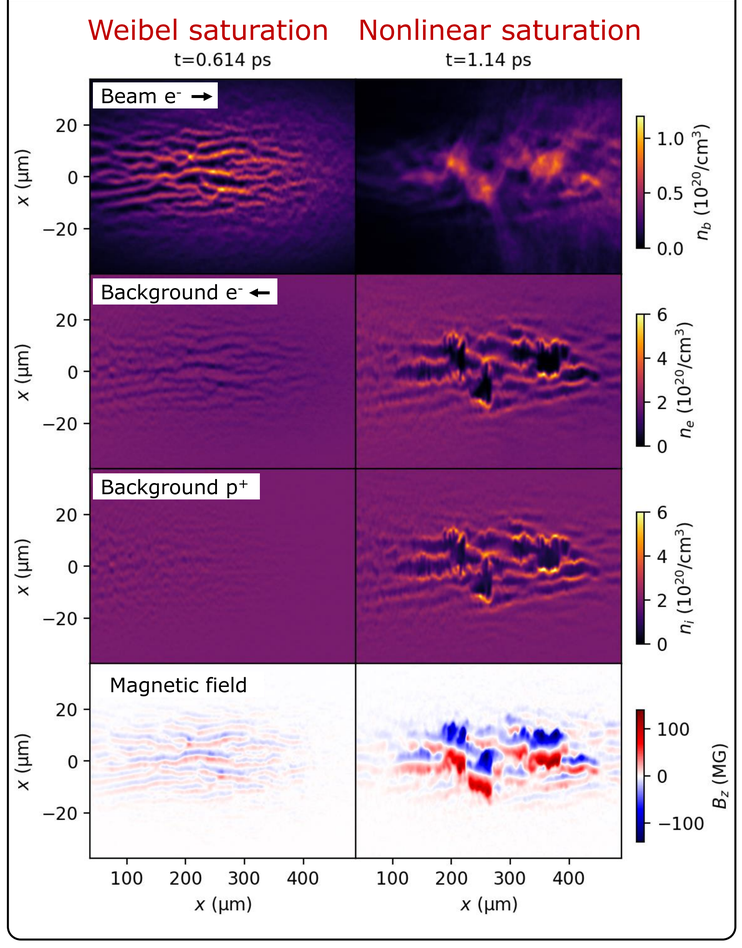
$$D \gg 17\ \mu\mathrm{m},$$
$$T \gg 470\ \mathrm{fs}.$$

We simulate a **5 μC beam** with **53 μm** diameter and **1.3 ps** length.

Possible experimental setup



Simulation with 5 μC LWFA e-beam



Future work

- Extend analysis to pair beams
- Simulate experimental conditions in 3D
- Explore effects on GRB evolution

Acknowledgements

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