Analysis of unsteady strong shock waves with applications to hydrodynamic instability growth

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Introduction

- Unsteady strong shock waves emerge and play a dominant role in stockpile stewardship, astrophysics, and fusion research [1].
- Classical shock theory fails to capture features of such waves that can have a significant effect on hydrodynamic instabilities [1,2].
- **Objective: Develop a method to semianalytically solve problems involving unsteady strong shock waves in order to understand the resulting hydrodynamic instability growth.**

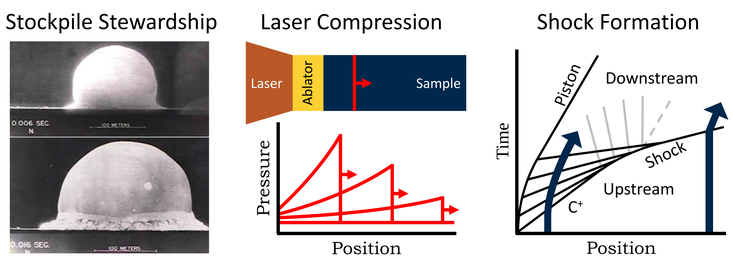

Fig 1: Stewardship (left), laser compression (center), and wave steepening (right) involve unsteady shocks.

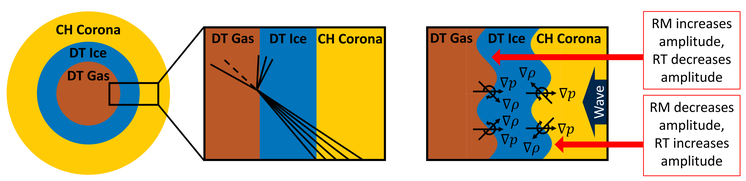

Fig 2: Some inertial confinement fusion schemes utilize compression waves that interact with interfaces [3].

Method

- The method of characteristics (MoC) reduces PDEs to ODEs along specific paths, enabling solutions to 1D nonlinear hydrodynamics.
- The Riemann problem describes a system after diaphragm release.
- The new method leverages the MoC with boundary conditions from the Riemann problem and a novel "pick-two" rule.

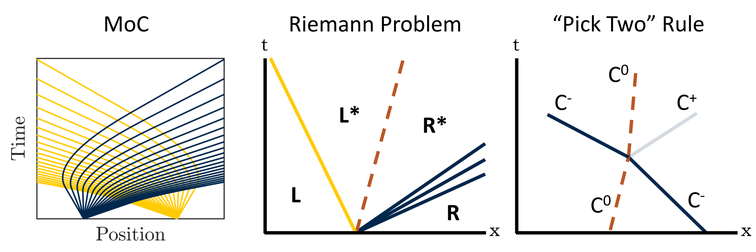

Fig 3: Schematics demonstrating the MoC (left), the Riemann problem (center), and the "pick two" rule (right).

Results: Wave Steepening Solution

- The method captures the wave steepening process and correctly calculates the reflected wave and variable entropy region.
- Simulations were performed using our in-house, high-order accurate discontinuous Galerkin (DG) code to verify the theory.

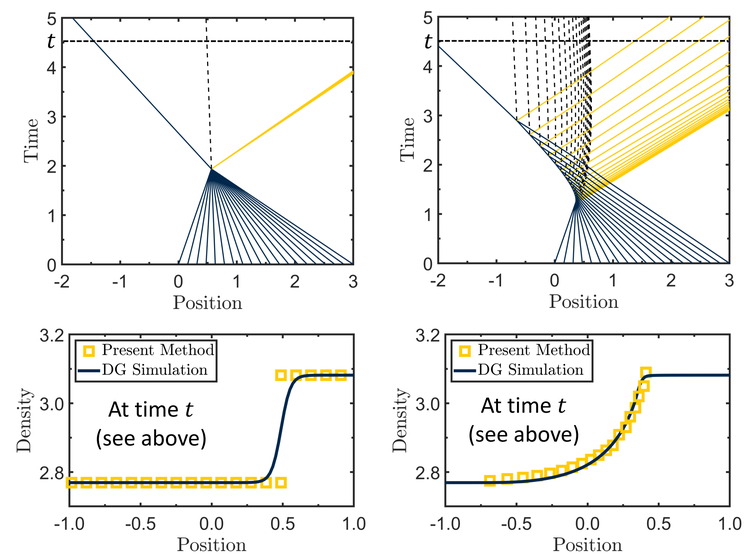

Fig 4: The MoC solution (top) for a steepening compression wave and the density profile through the emerging contact region (bottom) for a compression wave with a linear (left) and sinusoidal (right) velocity profile.

- The accuracy of the method is further verified by comparing the strengths of the waves that emerge from the steepening process to classical shock theory and the exact Riemann problem solution.
- The method tracks the solution even for strong compression waves.

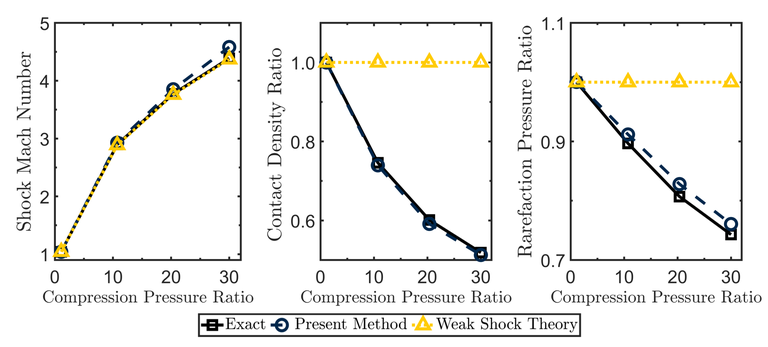

Fig 5: The strength of the shock (left), contact (center), and rarefaction (right) waves that emerge from the shock steepening process from classical theory, the exact solution to the Riemann problem, and the present method.

Results: Instability Growth

- The method is applied to explore perturbation growth along an interface accelerated by a compression wave.
- The proximity of the interface to the wave has critical implications for the type of growth experienced, ranging from pure Rayleigh-Taylor (RT) growth to pure Richtmyer-Meshkov (RM) growth.
- The interface growth is predicted using buoyancy-drag models with the interface acceleration history provided by the present method.

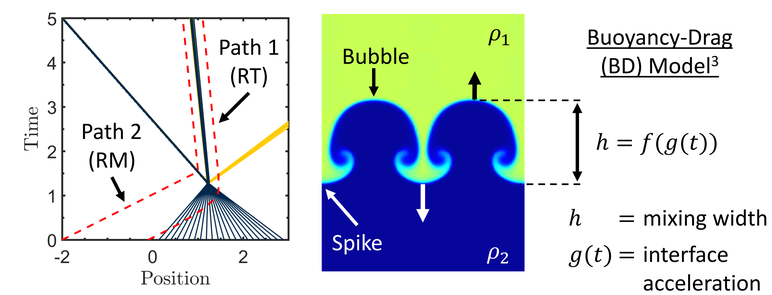

Fig 6: A compression wave steepening into a shock (left) and a diagram of perturbation growth features (right).

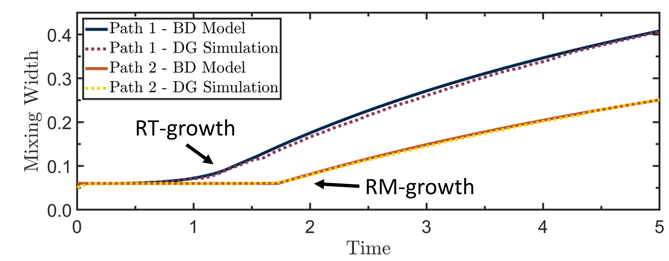

Fig 7: Perturbation growth from simulation and BD models for interfaces following the paths outlined in Fig 6.

Conclusions and Acknowledgement

- A semianalytical method for unsteady strong shocks was developed and applied to study hydrodynamic instability growth.
- The method correctly captures key features of unsteady strong shocks, including a variable entropy region and reflected wave.
- Perturbation growth resulting from an unsteady strong shock can result in a complex combination of RT and RM growth.

[1] R. Courant and K. Friedrichs, Springer-Verlag, Appl. Math. Sci. **21**, 1948. [2] A. Shapiro, The Ronald Press Company **11**, 1954. [3] D. Swift et al., Phys. Rev. E, **78**, 2008. [4] Y. Srebro et al., Laser Part. Beams, **21**, 2003.

This work is funded by the U.S. Department of Energy (DOE) NNSA Center of Excellence under cooperative agreement number DE-NA0003869. In addition, this work was supported in part by the DOE NNSA Stewardship Science Graduate Fellowship under grant DE-NA0003960. Furthermore, this work was supported in part by the Lawrence Livermore National Laboratory under contract DE-AC52-07NA27344 and subcontract B632749.