Ethnic Diversity, Public Spending and Political Regimes

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Abstract

Do democracies discriminate less against minorities as compared to non-democracies? How does ethnic diversity affect discrimination under various political regimes? We build a theory which tries to answer such questions. In our model, political leaders (democratically elected or not) decide on the allocation of spending on different types of public goods: a general public good and an ethnically-targetable public good which benefits the majority ethnic group while imposing a cost on the minorities. We show that, under democracy, higher ethnic diversity leads to greater provision of the general public good while lower diversity implies higher provision of the ethnically-targetable good. Interestingly, the opposite relation obtains under dictatorship. This implies that political regime changes can favour or disfavour minorities based on the incumbent level of ethnic diversity. Several historical events involving regime changes can be analysed within our framework and are consistent with our results.

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1 Introduction

Discrimination against minorities — ethnic, religious, linguistic, etc. — is a serious concern worldwide. Sometimes such discrimination takes a overt form via directed violence, forcible segregation (residential and or occupational). In many contexts this is more covert, working through discrimination in the labour market (manifest in hiring decisions, glass ceilings, etc.) or even through the public offices by provision of lower/inferior public goods (roads, infrastructure, health facilities, educational institutions, etc.). Such systematic exclusion of segments of the population is damaging not only from a normative perspective — there are potential economic inefficiencies arising out of this. The role of political institutions in determining various economic outcomes has received much attention in the recent years. Typically, democracies are perceived to be superior to non-democracies on many dimensions; particularly, on the allocation of public spending (see e.g., Tavares and Wacziarg (2001), Deacon (2009), Acemoglu et al. (2014)). So can the issue of discrimination against minorities be mitigated by superior institutional structures like democracy? In other words, is discrimination necessarily lower under democracies as opposed to dictatorships? Recent events suggest otherwise. Consider the recent surge in violence against the Muslim Rohingya community in Myanmar — this is after a democratically elected government assumed power. Therefore, can one pin down which factors might condition the degree of discrimination under different political regimes? In particular, how does ethnic diversity affect discrimination under various political regimes? Here, we put forward a tractable theory to answer such questions.

Our theory considers two alternative political regimes: democracy and dictatorship. In our model, the society is composed of a dominant ethnic group and an amalgamation of many other (minority) groups. Irrespective of the political regime, one of the main tasks of the government is to allocate public spending. Such spending has an important role to play in the economy, particularly in boosting output and economic growth. Political parties within a democracy would understandably take this spending seriously, as their terms in office would depend quite critically on this. For dictators, who are not elected through popular mandate, there is an alternative incentive to direct public spending in a certain way: they would typically embezzle a portion for themselves, while also ensuring that they minimise the chances of a popular uprising.

We introduce the notion of discrimination in this setting in the following manner: two kinds of public spending are possible in this society. The first type is on a “general” public good which benefits everyone irrespective of their ethnic background, while the other, an “ethnic”

\[2\text{Consider the centuries old “caste” system in India. Incidents of atrocities upon the lower castes are not uncommon even today whenever there is an alleged “transgression” of the boundaries by them.}\]

\[3\text{Persson (2002) contains an excellent overview.}\]

\[4\text{The increase in inter-ethnic cooperation in Rwanda under President Kagame’s quasi-autocratic rule (see Blouin and Mukand (2018)) also points in a similar direction. We shall turn to a detailed discussion of such events later.}\]

\[5\text{See, for instance, Barro (1990), Futagami et al. (1993), Turnovsky (1997), Ghosh and Roy (2004), etc.}\]
public good, benefits only the dominant ethnic group. This latter good is exclusionary and hence has a negative impact on the utility of the minorities.\(^6\) Hence, whenever there is a positive amount spent on the ethnic good, it is classified as discrimination in our setup — the greater the spending on the ethnic good, the higher the discrimination.\(^7\) There is, however, heterogeneity in the preferences for this “ethnic” good amongst the majority — some favour it more than others.

We first study a democratic setting with two parties which compete for the citizens’ votes by each promising budgetary allocations on the two public goods. Here, we show that the equilibrium allocation involves a monotonic relationship between ethnic diversity and the share of the general public good. Above a certain threshold level of ethnic diversity, the entire budget is spent on the general public good by either political party; below this threshold, the spending is entirely on the ethnic public good. This is intuitive, as in the absence of a “large” dominant group, political parties will strive to compete for votes from all sections of the population (and hence invest in the general public good), while in the presence of such a group, the parties would spend all of their energies in catering to that group (thereby investing in the ethnic good) even at the cost of antagonising the minorities.

In the case of a dictatorial regime, there is no explicit role for political parties. The dictator decides on the allocation of public spending with largely two considerations in mind: appropriation of the public funds (“rents”) and surviving any potential uprising by the citizens. In the eventuality of a successful revolt, there is a return to the two-party democratic regime and the dictator is disallowed from appropriating any amount of the public budget. Thus, the dictator has to factor in how the different ethnic groups will react – i.e., support a rebellion or not – when he makes his public spending allocation. Clearly, the decision by any citizen would depend upon what she thinks the alternative scenario (in this case, democracy) will deliver to her. What makes the issue perhaps more interesting is that what democracy delivers depends upon how ethnically diverse the society is. So our subgame perfect equilibria in the dictatorship game depend upon the level of ethnic diversity.

We show that when ethnic diversity is higher than a certain threshold, the dictator tilts spending entirely towards the dominant ethnic group.\(^8\) Also, when ethnic diversity is lower than that threshold, the dictator invests only in the general public good. In other words, in a very ethnically diverse society the dictator will actually only cater to the dominant ethnicity while neglecting the minorities. It is precisely an ethnically homogeneous society (one with a large dominant ethnic group) which will witness spending only on the general public good and hence no discrimination. Observe that this is completely contrary to the pattern of public spending under democracy.

\(^6\)Section 4.1 discusses the case of having another ethnic good which is favourable only to the minorities.

\(^7\)To be sure, this is a stylised view of the idea of discrimination. Nonetheless, this is the aspect which is salient through the actions of the government; hence, we think it is a relevant depiction.

\(^8\)This threshold is the same as the one where the switch in spending happens under democracy.
The intuition for this result is the following: with high diversity, the minority group has a strong incentive to rebel since they know that they will benefit from the general public spending in case the dictator is ousted and elections take place. So dissuading them is too costly for the dictator. In order to prevent members of the dominant group from joining the rebellion, targeted ethnic spending has to be offered to that group by the dictator. Conversely, with low ethnic diversity, the dominant group has an incentive to rebel since under democracy the entire spending will be directed towards them (complete discrimination). In this situation, the minority group will not rebel since democracy will not bring them any enjoyment from the public spending. Therefore, in order to dissuade some members of the majority from rebelling, a positive amount of only the general public good will be offered by the dictator. Discrimination is not optimal since under democracy the entire spending would be in favour of the dominant ethnic group. So providing the ethnic good would only prevent rebellion if the dictator did not appropriate any positive share of the budget. But clearly, that is sub-optimal from the dictator’s perspective. Hence the dictator tries to dissuade rebellion by committing to spend some positive amount on the general public good (and hence, no discrimination). Hence, the pattern of discrimination — particularly, how it varies with ethnic diversity — is strikingly different in a democracy as opposed to a dictatorship.

Moreover in our setup, the extent of appropriation is an endogenous choice variable for the dictator. This allows us to document the relationship between ethnic diversity and this level of appropriation by the dictator. The pattern is non-monotonic with a potential discontinuous jump at the threshold where the switch in spending (from ethnic targeting to general public spending) occurs. Our results provide a rationale — based on ethnic diversity — for why one observes a different pattern of discrimination and not just a different level of public spending in dictatorships as opposed to democracies. Additionally, our analysis suggests that as a fairly ethnically homogeneous society starts becoming even more homogeneous, the gap in the economic performance when under a dictatorship and a when under a democracy, starts shrinking (since the amount usurped by the dictator starts declining). In other words, it is for ethnically diverse societies where the formal institutional context matters more in terms of economic output.

Our theory can be used to interpret certain historical events like the changing nature of Hutu-Tutsi relations in Rwanda, the treatment of Chinese Indonesians during and after the Suharto regime and more recently the issue of Rohingya massacres in Myanmar. Each of these scenarios when viewed through the lens of our model appear to be consistent with the model’s predictions. We offer a more detailed treatment of these cases later.

The remainder of the paper is organised in the following way: Section 2 provides a discussion of the related literature. Section 3 develops the theory and presents the analytical results. Section 4 discusses some possible extensions, Section 5 contains some discussion regarding certain historical events in light of our theory and Section 6 concludes. All proofs are contained in the appendix.
2 Related Literature

By highlighting the connection between discriminatory public spending and political regimes within the context of ethnic diversity, our paper relates to various strands of literature. The link between ethnic diversity and public goods provision draws upon the recognition of the fact that when people are heterogeneous, so are their preferences, which thereby has an important bearing on how much and what sort of public goods are produced. For instance, the link between ethnic fractionalisation and public services is attributed to taste differences of different sections of the population (Alesina et al. (1999), Alesina and La Ferrara (2005)) and/or inability to impose social sanctions in ethnically diverse communities (Miguel and Gugerty (2005)), thus leading to failure of collective action. In most of this literature, the focus has been on coordination issues arising from taste diversity. The issue of how various minority groups fare from such public provision has largely been neglected.

A large section of the literature on discrimination against minorities deals with the evaluation of various corrective measures. These measures typically involve some form of earmarking or reservation of posts (often in public offices, educational institutions, etc.). Reserving political office for members from various marginalised groups has sometimes been found to be effective — in the sense of working in the interests of those groups (see e.g., Pande (2003), Chin and Prakash (2011) for evidence in the case of India where reservation has been in place for decades in favour of historically disadvantaged groups called the Scheduled Castes (SCs) and the Scheduled Tribes (STs)). There are other studies which suggest that the effects may be heterogeneous within the minorities (Mitra (2018)) or that they may not be persistent (Jensenius (2015), Bhavnani (2016)). But most of this literature is in the context of democracies; there is hardly any comparison with alternative political regimes. Also, these studies do not deal with how the political structure may be responsible for the existence of such discrimination in the first place.

Mukand and Rodrik (2015) make the distinction between electoral and liberal democracies where the former “are political regimes which allow political competition and generally fair elections, but exhibit considerable violations in the civil rights of minority and other groups not in power.” In their words, the main distinctive feature of a liberal regime is the presence of “the restraints placed on those in power to prevent discrimination against minorities and ensure equal treatment”. They develop a formal model to sharpen the contrast between electoral and liberal democracies and highlight circumstances under which liberal democracy can emerge. Their emphasis on distinguishing between different regimes (electoral and liberal democracies) in terms of the discrimination against minorities resonates with the main theme

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9Banerjee and Somanathan (2001), in studying the Indian districts, have suggested that more heterogeneous communities tend to be politically weaker, and therefore are likely to be denied the public goods of their choice and are more likely to get some of the inferior substitutes. See also Tajfel et al. (1971), Alesina and Drazen (1991), Alesina and Rodrik (1994), Alesina et al. (1999), Baldwin and Huber (2010) among others.
in our work. However, their focus is different from ours — they outline the conditions as to when liberal democracy may arise.

Padro-i-Miquel (2007) argues how it is possible for rulers who often extract enormous rents and grossly mismanage their economies to survive. This is possible in an environment where society is ethnically divided and institutions are weak. The issue of discrimination against minorities is not addressed in that framework. Burgess et al. (2015) find, in the context of Kenya during the 1963 – 2011 period, that those districts that shared the ethnicity of the president received twice as much expenditure on roads and almost five times the length of paved roads built relative to what would be predicted by their population share. This form of ethnic favouritism, which was evident during periods of autocracy, disappeared during periods of democracy in Kenya.

The above suggests that ethnicity of the ruler matters regarding the size and composition of public spending when it comes to dictators. Interestingly, the evidence from India suggests that something similar happens when rulers are popularly elected. Bardhan et al. (2008) find that the village councils with a leader from the scheduled castes (SC) or scheduled tribes (ST) tend to receive more credit from the Integrated Rural Development Programme (IRDP). Besley et al. (2004) finds that for high spillover public goods (such as the access road to a village), the residential proximity to the head of the Gram Panchayat matters. For low spillover goods, the underlying preference of the head mainly counts. This prevalence of ethnicity-based targeting even in democracies is also borne out by cross-country studies (see e.g., Franck and Rainer (2012), Kramon and Posner (2016), De Luca et al. (2018)). In our paper, the dictator is only interested in increasing their rent from the national pie, and we have abstracted away from any non-pecuniary payoffs (like favouring co-ethnics per se); in the context of democracy, the political parties are standard expected votes-share-maximisers. Our work is related to Deacon (2009) where the differing incentives of political leaders from different regimes (democracy/dictatorship) are discussed.

On the subject of whether or not the nature of spending is monotonic in ethnic diversity, our paper is close to Fernandez and Levy (2008). They show how diversity in preferences affects the basic conflict between rich and poor in a framework where people are heterogeneous both in preferences and in incomes, and in which political parties and party platforms are endogenous. The government both redistributes income and funds special-interest projects (e.g., local or group-specific public goods), all from proportional income taxation. Their analysis demonstrates that the effect of increased diversity is non-monotonic. They, however, do not consider non-democratic settings.

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10 See also Munshi and Rosenzweig (2015) for an examination of the role of local ethnic politics in provision of local public goods.

11 Deacon (2009) also provides robust empirical evidence on the asymmetries in public spending across the different political structures.

12 See also Lizzeri and Persico (2005) for comparison of expenditure in terms of efficiency when the number of competing candidates change. In a related vein, Mitra and Mitra (2017) examine the implications of more competitive elections on redistributive outcomes like inequality and find a strong link.
3 The Model

Here, we develop a simple model to capture the link between public spending (general versus discriminatory) and ethnic diversity under different political structures. This will enable a direct comparison of public spending patterns across a democracy and a dictatorship for any level of ethnic diversity. We begin with the analysis of a democratic setup.

3.1 Democracy

Here we will assume that there are two (exogenously given) political parties, $A$ and $B$ who compete for votes from the citizens. There is a unit mass of voters and for simplicity assume that there is one dominant ethnic group with mass $\lambda \in [1/2, 1)$.\footnote{One could think of this $\lambda$–group being composed of several smaller distinct ethnic groups, with some overlap in taste for a common local public good. More on this later.} Hence, a lowering of (rise in) $\lambda$ would correspond to an increase (decrease) in ethnic diversity. The mass $1 - \lambda$ could be composed of several different ethnic groups or just one ethnic group; it does not matter in our setup.

There is a budget which may be spent on providing the citizens with public goods.\footnote{Typically such public spending has the potential to raise output and overall growth.} Let the size of the budget be unity. Now, the budget could be spent on two different public goods. One is a truly 	extit{general} public good — call it $G$ — investment in which benefits all citizens equally. The other is an ethnic-specific good $E$, designed to benefit only members of the dominant ethnic group, i.e., the $\lambda$– group. We will assume that the members of the minority actually are harmed by the provision of $E$; this is because providing $E$ implies 	extit{discriminating against the minorities}. The budget constraint — for either of the two parties — is given by

$$\lambda e + g \leq 1.$$ 

We will denote party $j$’s platform by $(g_j, e_j)$ for $j = A, B$. The parties simultaneously propose platforms, and each party seeks to maximize its expected number of votes given the other party’s platform.

We assume that there is heterogeneity in preference for the ethnic-specific good $E$ within the dominant ethnic group. The payoffs to the voters are described below.

Take the case when $e > 0$. On being offered $(g, e)$, the payoff to a member of the $(1 - \lambda)$– group is $g - \psi$ where $\psi \geq 0$. Higher the value of $\psi$, the greater the disutility to this minority group member from discrimination. In a sense, this parameter $\psi$ captures the direct costs of discrimination to a minority group member. On the other hand, the payoff to a member $i$ of the $\lambda$– group is given by

$$u_i(g, e; \lambda) = g + e + \epsilon_i$$
where $\epsilon_i$ is drawn from a distribution with cdf $F$ independently for every $i$ in the $\lambda$–group. Also, $E[\epsilon] > 0$ and $f \equiv F' > 0$ everywhere on the real line. Moreover, let $f$ be symmetric and unimodal so that the mode is at $E[\epsilon]$. This implies $F(0) < \frac{1}{2}$. This re-iterates the fact that it is more likely for a member (of the dominant group) to actually have a positive realisation of $\epsilon$, than not. The linear payoff structure is not crucial here, any $w(g + e + \epsilon_i)$ for $w' > 0$ suffices.

In the case where $e = 0$, then the payoff to a member of the minority is simply $g$ as is the payoff to a member from the dominant ethnic group; therefore, there is no discrimination in this situation.

Observe that the ethnic-specific good $E$, with its element of taste-heterogeneity, easily lends itself to the following interpretations. One could think of different scenarios where the dominant ethnic group specialises (or has disproportionate shares) in a certain sector/industry. Hence, increasing investment in $E$ would by and large benefit most members of the group but not all; some might actually be hurt if their fortunes are tied to other sectors/industries. Alternatively, one could think of this $\lambda$–group as being composed of smaller ethnically distinct sub–groups who are united in their common affinity for $E$. So the ethnic good $E$ could be viewed as a kind of “compromise” local public good for this $\lambda$–group, where every member of the $\lambda$–group has a positive expected return from consuming $E$, which is equal ex ante.\textsuperscript{15} From the perspective of the minority citizens, $E$ is something they are excluded from; hence, it embodies discrimination against them.

Now we are in a position to analyse the equilibrium of this simple game and then study its dependence on ethnic diversity (as captured by $\lambda$). In fact, the following observation is a step in that direction.

**Observation 1.** There exists a $\lambda > \frac{1}{2}$, such that both parties proposing to spend the entire budget on the public good $G$ is the unique equilibrium for every $\lambda \in \left[\frac{1}{2}, \lambda\right]$.

The intuition behind the result stated in Observation 1 is the following. When the dominant ethnic group is only slightly larger than the rest (in particular, their size is close to $1/2$) it is not optimal from a party’s perspective to simply try to win their votes by providing only the ethnic good; all the more so, since not everyone within the dominant ethnic group actually likes the ethnic good. Moreover, with one party promising to spend the entire budget on the general public good, any platform by the other party which spends less than the entire budget on this general good will be rejected by all members of the minority. Therefore, both parties spend the entire budget on the general public good so as to enlist the support of both the ethnic groups.

This leads us to the question of what happens when the size of the dominant ethnic group is beyond this threshold level of $\lambda$. In particular, is there any other equilibrium other than

\textsuperscript{15}This aspect of an ethnic group having it’s own specific type of “local” public good is similar in spirit to Fernandez and Levy (2008).
when we move beyond $\lambda$? The following observation sheds some light on this matter.

**Observation 2.** There exists a $\lambda > 1/2$, such that both parties proposing to spend the entire budget on the ethnic-specific good $E$ is the unique equilibrium for every $\lambda \in [\lambda, 1)$.

The intuition behind the result stated in Observation 2 is the following. When the dominant ethnic group is truly large in size — in particular, say close to unity — then for a political party to ensure an electoral victory getting their support is enough. So the ethnic minority can be neglected as long as their size is sufficiently small. Given that a dominant proportion of the majority ethnic group actually *likes* the ethnic good (recall, $F(0) < \frac{1}{2}$), it becomes optimal for a party to just spend the entire budget on the ethnic good.

So far we have established that there are intervals $[1/2, \lambda]$ and $[\lambda, 1)$ where the equilibrium is unique though different in each of the two intervals. Moreover, it must be that $\lambda \leq \lambda$. This raises the following questions: are $\lambda, \lambda$ actually distinct? If yes, then what are the equilibria for any $\lambda \in (\lambda, \lambda)$?

It turns out that there is a unique value of $\lambda$ — call it $\lambda$ — such that $(e_A = e_B = 0, g_A = g_B = 1)$ is the unique equilibrium for all $\lambda < \lambda$ and $(e_A = e_B = 0, g_A = g_B = 1)$ is the unique equilibrium for all $\lambda > \lambda$. This is stated more formally in the following observation.

**Observation 3.** There exists a unique $\lambda \in (1/2, 1)$, such that $V(\lambda) \equiv \lambda\{1 - F((\lambda - 1)/\lambda)\}$ is lower (higher) than $1/2$ for $\lambda < (>)\lambda$ and $V(\lambda) = 1/2$.

Observation 3 pins down the unique threshold $\lambda$. Note, it depends *only* upon the distribution $F$. Using the equation defining $\lambda$, we can ask how changes to this distribution affects the position of $\lambda$ on the interval $[1/2, 1)$. In particular, it is clear from inspection that a rightward shift of the density function $f$ will lead to a lower $\lambda$. So any shift of the distribution $F$ in the sense of *first-order stochastic dominance* will result in a lower value of this threshold $\lambda$.

Intuitively, a smaller possibility of negative realizations of the $\epsilon$ shock and hence a higher value for $E[\epsilon]$ ($> 0$, by assumption) implies that the dominant group’s support may be sufficient for victory (by spending solely on the ethnic good) even for smaller values of $\lambda$.

Before proceeding any further, it may be of interest to know the nature of equilibrium platforms at $\lambda = \lambda$. It turns out (as noted in the following observation) that there are four equilibria in pure strategies for $\lambda = \lambda$. This multiplicity arises precisely from the fact that $V(\lambda) = 1/2$.

**Observation 4.** There exist four equilibria in pure strategies and a family of mixed strategy equilibria for $\lambda = \lambda$. Moreover, the provision of $G$ is either 0 or 1 depending upon which equilibrium is played out.

The observations above collectively yield the following result.
Proposition 1. In a democracy, the relationship between ethnic diversity (as captured by the magnitude of \( \lambda \)) and the share of the pure public good \( G \) (or alternatively, the ethnic-specific public good \( E \)) provided in equilibrium is (weakly) monotonic in \( \lambda \). In particular, there is unique value of \( \lambda \) — namely, \( \hat{\lambda} \) — such that for all \( \lambda < \hat{\lambda} \) the unique equilibrium allocation involves spending the entire budget on \( G \), and for all \( \lambda > \hat{\lambda} \) the unique equilibrium allocation involves spending the entire budget on the ethnic-specific public good \( E \). For \( \lambda = \hat{\lambda} \), the equilibrium provision of \( G \) is either 0 or 1.

Next we move on to a similar analysis when instead of a two-party electoral democracy, we have a dictatorship in place.

3.2 Dictatorship

In a dictatorship, there will be no explicit role for any political parties. The decision regarding the allocation of budgetary funds for investment into \( G \) and \( E \) will be taken by the dictator, whom we shall refer to as \( D \).

The other elements of the model remain just as before. We will have our dominant ethnic group of size \( \lambda \) and it will be assumed that the citizens have no direct control over the budget (just as before). In a democratic setup, the allocations proposed by the two parties were governed by considerations of support by the citizens through the ballot. Here, under a dictatorial regime, certain different considerations will impel the dictator \( D \) to allocate spending in a particular way. We will return to these considerations shortly.

There are some basic considerations which any dictator must take into account. First, there is always a threat of a mass revolution. Hence, our dictator \( D \) knows that with some chance he will not be ruling the roost in the near future. Secondly, staying in power is valuable to \( D \); this provides access to “rents” which depend on the public budget.\(^{16}\) For simplicity, we will assume the following: \( D \) lives for one period during which there is a chance of a mass revolution and if he survives the revolution (or if there is none) then he can usurp a part of the public budget. In case \( D \) is overthrown, he gets a zero payoff.

Now this brings us to the question of what determines the incidence and success of a “revolution”. We posit a simple two-stage game to capture the idea of a “revolution”. In the first stage, the dictator proposes an allocation \((g_D, e_D) \geq (0, 0)\) and also his “share” \( \mu \) of the budget.\(^{17}\) The allocation \((g_D, e_D)\) is subject to feasibility constraints. Therefore,

\[
g_D + \lambda e_D \leq 1 - \mu.
\]

In the second stage, the members of the two different ethnic groups simultaneously decide

\(^{16}\)More on these “rents” later.

\(^{17}\)Since the citizens know the size of the budget, announcing \((g_D, e_D)\) is sufficient for the citizens to infer \( \mu \).
whether or not to revolt against $D$. Formally, each citizen chooses an action from the set \{R, NR\} where $R$ denotes revolt and $NR$ not revolt. This action is taken individually by each citizen — hence, no coordination issues — and is done after each $\lambda$–group citizen draws her realization of $\epsilon$ which is the stochastic component of the payoff from $E$. This means that the choice of revolting or not is made after she knows her exact valuation of the $E$–good.

What happens when the revolt is “successful” and $D$ is deposed? We take the position that a two-party democracy emerges at the conclusion of a successful rebellion. The idea is that the political parties can be thought to remain dormant under a dictatorship, but emerge once the dictator loses power. In reality, in countries which move back-and-forth between democracy and (military) dictatorships, prominent political parties are quite resilient and resume activities soon after the dictator is deposed (see e.g., the political histories of Pakistan and Zimbabwe among others).

At the end of the period, exactly one of the two things happen:

(i) all citizens choose $NR$ or some choose $R$ but the revolt is unsuccessful and $D$ implements his proposed $(g_D, e_D)$ and usurps $\mu$.

(ii) The revolt results in $D$'s removal and democracy is restored. Under democracy, we have the citizens voting and deciding the allocation of the budget via the ballot.\footnote{Given that the outcome of the revolution (if there is one) is probabilistic, it is not possible for the political parties to gain any further information on any of the individual citizens’ realisations of $\epsilon$; note, they already know the distribution $F(.)$ of these $\epsilon$ variables. Hence the possibility of any type of Bayesian updating does not exist in this setup.}

See Figure 1 for a graphical depiction of the timing.

Let $p$ denote the probability of a successful revolution. How does $p$ depend on the parameters of the model? We assume that larger the size of the rebel group, the higher is $p$. For the sake of concreteness, let $p$ equal the mass of people who choose action $R$. As a tie–breaking rule, we have that whenever a citizen is indifferent between $D$'s offer and the alternative equilibrium allocation under democracy, she chooses $NR$. This is easily justified by assuming there is...
a fixed cost $c \geq 0$ which is incurred by the citizen in case she chooses to rebel. In fact, we could explicitly incorporate this (private) cost of revolution $c > 0$ into our model. However, we refrain from doing so as it complicates notation without adding any further insights; all our qualitative results are unchanged as long as $c$ is sufficiently low.\footnote{Acemoglu et al. (2010) study how non-democratic regimes use the military (which consists of a set of individuals who act in their own self-interest), and how this can lead to the emergence of military dictatorships (when the military decide that turning against, rather than aligning with, the elite would enable them to pursue their own objectives). We abstract from such dynamic considerations and focus on public spending patterns under alternative regimes.}

In principle, $D$ can set $\mu$ equal to unity. That implies $(g_D, e_D) = (0, 0)$. In a dynamic setting, this would never transpire in equilibrium as then output which presumably depends upon public spending will drop to zero. This implies that the budget available for the subsequent period — which depends on this period’s output — will be 0. Hence, any $D$ with a sufficiently high discount factor will not set $\mu = 1$. Additionally, one can imagine that in such a scenario there would be hue and cry internationally, there would be a humanitarian crises as a result of which $D$ might be removed from office. Therefore, we set an upper bound on $\mu$ — call it $\overline{\mu}$ — which is below unity. So any $\mu > \overline{\mu}$ results in the dictator’s immediate removal (i.e., $p = 1$ for $\mu > \overline{\mu}$). Of course, $\overline{\mu}$ can be arbitrarily close to 1.

We solve this two–stage game backwards, as is standard practice. $D$‘s problem in the first stage is the following:

$$
\begin{align*}
\max_{(g_D, e_D, \mu)} & \quad p \cdot 0 + (1 - p) \mu \\
\text{s.t.} & \quad g_D + e_D \lambda \leq 1 - \mu
\end{align*}
$$

where $\mu \in [0, \overline{\mu}]$. The optimal choice of $(g_D, e_D, \mu)$ clearly depends upon the degree of ethnic diversity $\lambda$, not just through the budget constraint but also via $p$.

Take any given $\lambda \in [1/2, 1)$. Recall the cutoff value of $\lambda$, namely $\hat{\lambda}$, from the democratic setup. What the citizens can expect to transpire in democracy will depend on where $\lambda$ stands in relation to $\hat{\lambda}$ (see Proposition 1).

We start with $\lambda < \hat{\lambda}$.

Case 1: $\lambda < \hat{\lambda}$.

Take a voter from the minority group, i.e. from the $(1 - \lambda)$– group. If $D$ is removed (and voting takes place under democracy), he gets a payoff of 1 since the entire budget is spent on $G$ for $\lambda < \hat{\lambda}$ (see Proposition 1). On the other hand, if $D$ stays in power, then he gets a payoff of $g_D$ (or $g_D - \psi$ if $e_D > 0$). Hence, a member of the minority group will choose $R$ whenever $g_D < 1$. So to prevent the $(1 - \lambda)$– group from rebelling, $D$ must choose $\mu = 0$ and get a payoff of 0. As we show below, this is not optimal for $D$. In fact, $D$ will choose $\mu > 0$ and all of the $(1 - \lambda)$– group will choose $R$.

Now take a citizen $i$ from the $\lambda$–group. If $D$ is removed, he gets a payoff of 1. On the other
Figure 2: Payoffs to citizens. The possible payoffs to each group of citizens under dictatorship depending upon the success/failure of the revolution for $\lambda < \hat{\lambda}$.

hand, if $D$ stays in power, then he gets a payoff of $g_D + e_D + \epsilon_i$, if $e_D > 0$; otherwise he gets $g_D$. So this citizen $i$ will choose $R$ only if one of the following is true: $e_D = 0$ and $g_D < 1$ or $g_D + e_D + \epsilon_i < 1$. Like in the case of the $(1 - \lambda)$- group, setting $g_D = 1$ prevents revolution. In fact, all citizens choose $NR$. However, this also guarantees $\mu = 0$ and hence a payoff of 0 for $D$. As argued below, $D$ will choose $\mu > 0$ and set $e_D > 0$.

See Figure 2 for a tabular representation of the citizens’ payoffs in this scenario.

The corresponding expression for $p$ (for $e_D > 0$) is given by

$$p = 1 - \lambda + \lambda[Prob.(g_D + e_D + \epsilon_i < 1)] = 1 - \lambda + \lambda F(1 - g_D - e_D).$$

Hence, $D$’s problem can be written as:

$$\max_{g_D, e_D, \mu \in [0, \pi]}[1 - F(1 - g_D - e_D)]\mu \lambda$$

subject to

$$g_D + e_D \lambda \leq 1 - \mu$$

$$g_D, e_D \geq 0.$$  

Given that the budget constraint is binding, the problem becomes:

$$\max_{g_D, e_D}[1 - F(1 - g_D - e_D)](1 - g_D - \lambda e_D)$$

subject to

$$g_D, e_D \geq 0.$$  

Setting up the standard Lagrangean FOCs makes it clear that $g_D = 0$ and $e_D > 0$. So we
can write the objective function as \([1 - F(1 - e_D)](1 - \lambda e_D)\) where \(e_D \in [\frac{1 - \mu}{\lambda}, \frac{1}{\lambda}]\). So the optimal \(e_D\) is either \(\frac{1 - \mu}{\lambda}\) or lies in \((\frac{1 - \mu}{\lambda}, \frac{1}{\lambda})\).

For an interior solution, \(\text{FOC}(e_D): (1 - \lambda e_D)f(1 - e_D) = \lambda [1 - F(1 - e_D)]\).

Before proceeding any further, we introduce some mild restrictions on the distribution function of \(\epsilon\), namely, \(F(\cdot)\).

**Assumption A1:** \(\frac{f(x)}{1 - F(x)}\) is (weakly) increasing in \(x\).

**Assumption A2:** \(f(x) \geq f'(x) \forall x < 0\).

**Assumption A3:** \(f(0) \leq 1/4\).

**Assumption A4:** \(f'(1) \geq 0\).

These assumptions essentially require that the spread of \(\epsilon\) is relatively “smooth” in the sense that there is non-negligible mass in the range away from 0. Moreover, most standard distribution functions (e.g. logistic distribution) satisfy them. To take a specific example, consider the general logistic distribution \(F(\cdot)\) of the variable \(X\) where \(X = a + bZ\) and \(Z\) follows the standard logistic distribution. All \(a \geq 1\) and \(b \geq 1\) satisfy A1–A4.

A1 guarantees that there is a unique interior solution to the FOC w.r.t. \(e_D\). Call it \(e^*_D\). Moreover by A4, we also have that the second-order condition w.r.t. \(e_D\) is negative at \(e_D = e^*_D\). Thus, we have that \(e^*_D\) is a maxima.

We also have \(\frac{\partial e^*_D}{\partial \lambda} < 0\) which means that were \(D\) to remain in power (no revolution or an unsuccessful one) then greater the diversity, the higher the public investments. This is more formally stated in the following observation.

**Observation 5.** For \(\lambda < \hat{\lambda}\), the dictator \(D\) will offer to provide a positive of amount of only \(E\). Moreover, \(\frac{\partial e^*_D}{\partial \lambda} < 0\).

The intuition behind the result in Observation 5 can be found from noting the following. When diversity is relatively high (\(\lambda\) is lower than the threshold \(\hat{\lambda}\)), then a transition to democracy results in the provision of only the \(G\) good; the entire budget is spent on it. This clearly is the best possible situation for the minority (the \((1 - \lambda)\) group). Hence, they will always revolt for \(\lambda < \hat{\lambda}\). Among the majority (the \(\lambda\) group), not everyone would revolt as long as a positive amount of \(E\) and/or \(G\) is provided given the heterogeneity in the preference for \(E\). Moreover, providing \(E\) is “cheaper” as it can be more precisely targeted at the majority (the \(\lambda\) group). As the size of this \(\lambda\) group increases (a reduction in diversity), the threat from the minority becomes less important and hence the dictator can economize on the provision of \(E\); this explains the negative sign of \(\frac{\partial e^*_D}{\partial \lambda}\).

We turn to the next scenario.

**Case 2:** \(\lambda > \hat{\lambda}\).

Take a voter from the minority group, i.e. from the \((1 - \lambda)\) group. If \(D\) is removed (and
Figure 3: Payoffs to citizens. The possible payoffs to each group of citizens under dictatorship depending upon the success/failure of the revolution for $\lambda > \hat{\lambda}$.

voting takes place under democracy), he gets a payoff of $-\psi$ since the entire budget is spent on $E$ for $\lambda > \hat{\lambda}$ (see Proposition 1). On the other hand, if $D$ stays in power, then he gets a payoff of $g_D$. Hence, a member of the minority group will always choose $NR$ since $g_D \geq 0 \geq -\psi$.

Now take a citizen $i$ from the $\lambda$–group. If $D$ is removed, he gets a payoff of $1/\lambda + \epsilon_i$. On the other hand, if $D$ stays in power, then he gets a payoff of $g_D + e_D + \epsilon_i$, if $e_D > 0$; otherwise he gets $g_D$. Note, this citizen $i$ will definitely choose $R$ if $e_D > 0$ since $1/\lambda > g_D + e_D$ (this follows from $g_D + e_D \lambda \leq 1 - \mu$ and $\mu \geq 0$). However, if $e_D = 0$ and $g_D > 0$ then this citizen chooses $R$ only if $1/\lambda + \epsilon_i > g_D$.

See Figure 3 for a tabular representation of the citizens’ payoffs in this scenario.

We claim that $D$ will always offer only $G$ in this scenario.

**Observation 6.** For $\lambda > \hat{\lambda}$, the dictator $D$ will offer to provide a positive of amount of only $G$, i.e., $e^*_D = 0$ and $g^*_D > 0$.

In light of Observation 6, we can express the revolt success probability as

$$p = \lambda[Prob.(1/\lambda + \epsilon_i > g_D)] = \lambda[1 - F(g_D - (1/\lambda))].$$

Hence, $D$’s problem can be written as:

$$\max_{g_D \geq 1 - \mu, \mu \in [0, \mu]} \{1 - \lambda[1 - F(g_D - (1/\lambda))]\}\mu$$

subject to

$$g_D \leq 1 - \mu.$$
simply in terms of $\mu$. Hence, $D$ chooses $\mu \in [0, \bar{\mu}]$ to maximise the following:

$$\{1 - \lambda[1 - F(1 - \mu - (1/\lambda))]\} \mu$$

Differentiating the above expression w.r.t. $\mu$ yields

$$1 - \lambda[1 - F(1 - \mu - (1/\lambda))] - \mu \lambda f(1 - \mu - (1/\lambda)) \equiv \theta(\mu, \lambda).$$

Notice, that $\theta(\mu, \lambda) > 0$ for $\mu = 0$. Hence, the optimal $\mu$ must either be $\bar{\mu}$ or in the interior i.e., in the interval $(0, \bar{\mu})$.

For an interior solution, $\text{FOC}(\mu)$ is given by $1 - \lambda[1 - F(1 - \mu - (1/\lambda))] = \mu \lambda f(1 - \mu - (1/\lambda))$.

Observe the FOC w.r.t. $\mu$. It is clear from inspection that the LHS exceeds the RHS at $\mu = 0$. Moreover, the LHS is decreasing in $\mu$. Assumption A2 implies that the RHS is increasing in $\mu$. Thus, a unique intersection is guaranteed giving us an unique solution to the FOC condition. Call this solution $\mu^*$. Additionally, the very same assumption A2 guarantees that the second–order condition for a maxima is satisfied.\(^{20}\)

Clearly, depending upon the size of $\lambda$ the optimal choice of $\mu$ by $D$ may vary between an interior one and that of $\bar{\mu}$. Without imposing further structure on $F(\cdot)$, it is not possible to guarantee an interior choice of $\mu$ for every $\lambda > \hat{\lambda}$. However, one can make the following claim about interior optima in this scenario: if for some $\lambda$, $D$ (optimally) chooses an interior $\mu$, then $D$ continues to do so for all higher values of $\lambda$. This is stated more formally below.

**Observation 7.** Suppose that for some $\tilde{\lambda} \in (\hat{\lambda}, 1)$, $D$’s optimal choice of $\mu$ lies in the open interval $(0, \bar{\mu})$. Then for every $\lambda \in (\tilde{\lambda}, 1)$, $D$’s optimal choice of $\mu$ will be strictly smaller than $\bar{\mu}$.

Now we ask the question: how does the provision of $G$ change as we change ethnic diversity, as captured by $\lambda$? Will $D$ choose to appropriate more or less as $\lambda$ changes? In other words, what is the sign of $\frac{\partial \mu^*}{\partial \lambda}$?

The following observation provides the answer.

**Observation 8.** For $\lambda > \hat{\lambda}$, either one of the following things happen in equilibrium:

(i) $D$ sets $g_D = 1 - \bar{\mu}$ for every $\lambda > \hat{\lambda}$. So $\mu^* = \bar{\mu}$ always.

(ii) There exists a threshold level $\tilde{\lambda} \in (\hat{\lambda}, 1)$ such that $D$ chooses $g_D > 1 - \bar{\mu}$ for every $\lambda \geq \tilde{\lambda}$. Here, $D$ will offer higher and higher amounts of $G$ as ethnic diversity falls (i.e., $\lambda$ increases). In other words, we have $\frac{\partial \mu^*}{\partial \lambda} < 0$.

\(^{20}\)Straight-forward differentiation yields that

$$\text{SOC}(\mu) : -\lambda[2f(1 - \mu - (1/\lambda)) - \mu f'(1 - \mu - (1/\lambda))]$$

which is negative under assumption A2.
The intuition behind the results in Observations 6 — 8 is the following. When diversity is relatively low (λ is higher than the threshold \( \hat{\lambda} \)), then a transition to democracy results in the provision of only the \( E \) good; the entire budget is spent on it. This clearly is the worst possible situation for the minority group as they do not value the \( E \) good at all. Hence, they will never revolt for \( \lambda > \hat{\lambda} \). Among the majority (the \( \lambda \)-group), provision of a positive amount of \( E \) by the dictator would invoke revolution *en masse* as long as the dictator steals (sets \( \mu > 0 \)). So the dictator provides no \( E \) and a positive amount of \( G \). This way not everyone in the majority group would revolt owing to the heterogeneity in the preference for \( E \). As the size of this \( \lambda \) group increases (a reduction in diversity), the threat from the majority becomes more important and hence the dictator increases the provision of \( G \) (which is synonymous with reducing \( \mu^* \)); this explains the negative sign of \( \frac{\partial \mu^*}{\partial \lambda} \).

Finally, we turn to the remaining possibility.

*Case 3: \( \lambda = \hat{\lambda} \).*

Before proceeding to the analysis for dictatorship in this case, it will be useful to recall the equilibrium outcome under democracy. Observation 4 informs us about the multiplicity of equilibria and that the equilibrium provision could range from \( g = 1 \) and \( e = 0 \) to \( g = 0 \) and \( e = 1/\lambda \). So the analysis here necessarily involves imposing some structure on the beliefs of the players as to what outcome will result in democracy given the infinite number of possible equilibria. Although, this would be an interesting exercise *per se*, we abstain from a complete treatment here and just analyze two possible belief structures by the players.

First, suppose that the citizens and the dictator believe that in case of a two-party competition (under \( \lambda = \hat{\lambda} \)), both parties will actually offer \( g = 1 \). The other belief structure that we will deal with is the polar opposite — namely, that the citizens and the dictator believe that (in the same scenario) both parties will actually offer to spend the entire budget on \( E \). Note, when we impose the belief (on the players) that the entire budget would be spent on \( E \), the case boils down to the scenario of \( \lambda < \hat{\lambda} \) (Case 1 above). On the other hand, when we impose the belief that the entire budget would be spent on \( E \), the case reduces to the scenario of \( \lambda > \hat{\lambda} \) (Case 2 above). In either case, we know what happens in the equilibrium under dictatorship.

This brings us to the issue of discontinuity (of several variables) at \( \lambda = \hat{\lambda} \). First, there is a switch from spending solely on \( E \) to spending solely on \( G \) as we pass this \( \lambda \) threshold. Secondly, one may wonder about the amount of expropriation by the dictator, namely, the fraction \( \mu^* \). How does that change around \( \lambda = \hat{\lambda} \)?

It turns out that the answer to this question is not straightforward. In particular, it is *not* possible to say if \( \mu^* \) is continuous in \( \lambda \) at \( \lambda = \hat{\lambda} \). It would depend upon the distribution \( F \).

\(^{21}\) Clearly, one could perform a more general analysis where all concerned players assume some probability distribution over the possible equilibrium outcomes. While certainly interesting, we believe that it would add little to the main arguments in this paper.
Figure 4: Appropriation under Dictatorship. The amount spent on $G$ and $E$ under dictatorship $(1 - \mu)$ for different levels of ethnic diversity ($\lambda$) is recorded on the vertical axis. Note the change in slope near the threshold $\hat{\lambda}$.

Specifically, on where the mean of the distribution is, by how much it exceeds 0.\textsuperscript{22}

However, the issue of the discontinuity does not cloud the main insights from the dictatorship analysis. This concerns the sharp change in the pattern of public spending on the $G$ and $E$ goods as one passes the $\hat{\lambda}$ threshold and also how the pattern varies within the $(1/2, \hat{\lambda})$ interval and also within the $(\hat{\lambda}, 1)$ interval.

The above discussion can be summarized in the following proposition.

**Proposition 2.** In a dictatorship, the relationship between ethnic diversity (as captured by the magnitude of $\lambda$) and the share of the pure public good $G$ offered by the dictator is (weakly) monotonic in $\lambda$. Specifically, the unique equilibrium allocation involves the dictator offering only $E$ for $\lambda < \hat{\lambda}$. For all $\lambda > \hat{\lambda}$, only $G$ is offered in equilibrium. Moreover, the provision of $G$ (weakly) increases as $\lambda$ increases beyond $\hat{\lambda}$.

We would like to draw attention to the proposition above and contrast it with our main result for the case of the democratic setup. Recall that in our democratic setup (with the standard two–party competition framework), we obtained that for a highly (ethnically) homogeneous society, the entire budget will be spent on providing $E$ — hence, maximum discrimination.\textsuperscript{23}

On the other hand, we find that for a dictatorship a highly (ethnically) homogeneous society will see a provision of only the $G$ good — therefore, no discrimination. Moreover, the provision of $G$ is (weakly) higher, *ceteris paribus*, the higher the degree of homogeneity.

\textsuperscript{22}This is clear from comparing the first–order conditions written in terms of $\mu$ for Cases 1 and 2. We omit the details of the calculations for the sake of brevity.

\textsuperscript{23}Interestingly, the more homogeneous the society in terms of preference for $E$ (as captured by the magnitude of $\lambda$), the higher the chance that the allocation of the budget is discriminatory (and potentially inefficient). This is partly driven by the fact that political parties need compete only for the votes of the ethnic majority as long as the latter are of sufficient numerical strength.
What is striking is the *reversal* in the pattern of spending under a dictatorship as compared to that in a democracy.

In Figure 4, we have depicted the discontinuity at $\lambda = \hat{\lambda}$ in terms of the spending by $D$. Specifically, we have shown that $\mu$ falls immediately on crossing $\hat{\lambda}$ from the left. Of course, this need not necessarily be the case. $\mu$ could also rise but we have settled on this depiction just for ease of exposition. Also, in the interval $(1/2, \hat{\lambda})$ we have $\frac{\partial e^*_D}{\partial \lambda} < 0$ which does not really inform about how $\mu$ changes with $\lambda$ since $\mu = 1 - \lambda e^*_D$; again, for the sake of exposition we have drawn this as downward-sloping. Finally, there could be values of $\lambda$ on either side of $\hat{\lambda}$ where $\mu$ is optimally chosen to equal $\bar{\mu}$. What is certainly fixed is that in the situation for $\lambda > \hat{\lambda}$ where $\mu$ is optimally chosen to be below $\bar{\mu}$, the line $(1 - \mu)$ starts rising from then on (see part (ii) of Observation 8).

Using this figure, one can attempt to pinpoint the level of ethnic diversity where the extent of appropriation is the lowest. Start with the interval $(\hat{\lambda}, 1)$. Aside from the case where $\mu = \bar{\mu}$ over that entire interval, it is clear that $D$ usurps lesser and lesser as $\lambda$ approaches 1; in other words, it is ethnically *homogeneous* societies which face the lowest amount of exploitation by a dictator. The comparison with the interval to the left of $\hat{\lambda}$ is difficult owing to the possible “jump” at $\lambda = \hat{\lambda}$.

4 Some Extensions

Here we discuss some implications of our theory by extending our model in certain directions.

4.1 An ethnic good for the minorities

There is an important asymmetry in the baseline model — the dominant ethnic group is allowed directed spending but the only benefit accruing to the minorities comes from general public spending. The reason behind this asymmetry was to precisely bring out the idea of discrimination as starkly (and as simply) as possible. However, it is possible to allow for a similar ethnic “good” for the minorities and yet retain the main results.

Suppose there exists an ethnic good $E'$ spending on which benefits only members from the minority ($1 - \lambda$ of the population). Also, let there be heterogeneity within the minority as to the preference for this good in a manner analogous to the majority’s ethnic good $E$. So when $E'$ is provided, more than half the minority group gets a positive realisation of the taste shock.

Before proceeding any further, it is important to clarify what “discrimination” means in

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24 There is in fact the possibility that $\mu = \bar{\mu}$ over the entire interval $[1/2, 1)$. This is clearly an uninteresting case although permissible theoretically. Nonetheless, the switch from $E$ to $G$ still occurs at $\hat{\lambda}$. 
this context. Clearly, when all the spending is on the general public good $G$, then there is no discrimination (like in the baseline model). However, when there is some spending on $E$, the minority feel discriminated against. Similarly, when there is some spending on $E'$, the majority group feels discriminated against. So now, we ask the question as to how both forms of discrimination fare under the different political regimes.

As before, first consider the situation under democracy. When the size of the dominant group is not too ‘large’ (similar to Observation 1), one can argue that only $G$ is provided as long as the disutility from discrimination ($\psi$) is sufficiently high in society. Now, the clear contender to this allocation is some combination of spending on both $E$ and $E'$. Notice however, that when spending is promised on both these ethnic goods, members from both groups feel discriminated and hence would simply prefer a platform with only general public spending (no discrimination). The logic in the case of low ethnic diversity (as in Observation 2) is unchanged from before — both parties will only cater to the dominant ethnic group when the latter is of a sufficient large size. Hence, the equilibrium allocations do not change under democracy.

Now moving over to the dictatorship scenario, we see that for high ethnic diversity (as in Observation 5) the dictator now has the option of using $E'$ as a means to dissuading the minority group. However, the more he spends on $E'$ the less he has for spending on $E$ and for his own consumption. Given that the minority group has a particularly high incentive to rebel, it is too costly to provide $E'$ by cutting back on $E$. Moreover, spending (sufficiently) on $E'$ at the cost of $E$ would mean that the dictator would lose some of the majority group citizens to gain a few minority votes; this is clearly not optimal from the dictator’s perspective. Thus, once again the dictator would use only $E$ in this situation. When ethnic diversity is low (as in Observation 6), the dictator will not spend on $E'$ since the minority would not be rebelling in any case.

In sum, it is possible to introduce this additional aspect of ethnic spending for the minorities without affecting the main findings in any significant manner.

### 4.2 Public Spending and Growth

The nature of public spending in an economy has the capacity to affect economic performance and in particular, output.

One can think of overall output $Y$ being a standard CES function — involving $g$, $\lambda$ and $e$ — of the following form:

$$Y(g, \lambda e) = \chi[\alpha g^\rho + (1 - \alpha)(\lambda e)^\rho]^{1/\rho}$$

where $\rho \in (0,1)$ and $\alpha \in (\frac{1}{2},1)$. We take the position that growth is mainly driven by investment in general public goods rather than in (ethnically) targetted goods. So we have
α in the interval \((\frac{1}{2}, 1)\). This guarantees that when the entire budget is being spent on either \(G\) or \(E\), spending it on \(G\) yields a higher output; i.e., \(Y(1, 0) > Y(0, 1)\).\(^{25}\) In this restricted sense, investment in the general public good outperforms investment in the ethnic–specific good in terms of overall output. \(\chi\) is the TFP term which we assume satisfies the following condition: \(\chi \geq \frac{1}{(1-\alpha)^\rho}\). Hence, it is possible to generate an output level of 1, when all the budget is spent on \(E\).\(^{26}\)

Our model can be used to examine if the relationship between ethnicity and growth is at all shaped by the existing political regime. As Propositions 1 and 2 state, the variation in the pattern of expenditure (between \(G\) and \(E\)) over the level of ethnic diversity (proxied by \(\lambda\)) is completely different under the two political regimes. From this perspective, our model delivers that as ethnic diversity increases in a democracy (crossing the \(\hat{\lambda}\) threshold from the right) there is an increase in output. However, under dictatorship we have that any increase in ethnic diversity (in the region to the right of \(\hat{\lambda}\)) leads to a potential fall in output.\(^{27}\)

Note however, to the left of the \(\hat{\lambda}\) threshold we have that any increase in ethnic diversity has no effect in a democracy while the effect is ambiguous under dictatorship. This suggests a convergence in output levels across the two institutional regimes for high levels of ethnic diversity is not improbable. So starting from \(\lambda = 1\), one should observe a divergence in output levels across regimes as ethnic diversity rises up to a point (the \(\hat{\lambda}\) threshold), beyond which increases in ethnic diversity may lead to a convergence in output levels across these two institutional regimes. Insofar we treat these output levels in our static framework as some steady state levels in a dynamic setting, our predictions can be interpreted in terms of output growth rather than levels.

### 4.3 Public spending and Welfare

The issue of citizen payoffs from public spending is directly present in our model. This makes it possible to analyse the welfare accruing to each ethnic group and overall under each of the two regimes, for varying levels of ethnic diversity. Of course, we are only capturing the welfare which follows directly from the utilisation of the public spending; the indirect effects which potentially arise due to a higher/lower output do not enter the current analysis.

First, consider the welfare under democracy. For the dominant ethnic group, under democracy there is always a positive level \(ex\ ante\) regardless of the degree of diversity.

Start with \(\lambda < \hat{\lambda}\). Here both the ethnic groups have the same welfare since the entire budget is spent on \(G\). So,

\[
U_\lambda = U_{1-\lambda} = 1.
\]

\(^{25}\)Note, \(Y(1, 0) = \chi\alpha^{1/\rho}\) and \(Y(0, 1) = \chi(1 - \alpha)^{1/\rho}\). Hence, \(\frac{Y(1, 0)}{Y(0, 1)} = \left(\frac{\alpha}{1-\alpha}\right)^{1/\rho} > 1\) since \(\alpha > \frac{1}{2}\).

\(^{26}\)This automatically guarantees that it is possible to get an even higher output by spending all the budget on the \(G\) good.

\(^{27}\)At least output cannot rise (by Observation 8).
Next consider $\lambda > \hat{\lambda}$. Here only the dominant ethnic group enjoys a non-zero payoff since only the $E$ good is provided. So the welfare to the dominant group as a whole is given by:

$$U_\lambda = \int \left( \frac{1}{\lambda} + \epsilon_i \right) dF = \frac{1}{\lambda} + E[\epsilon] > 1$$

where the last inequality follows from noting that $\lambda < 1$ and $E[\epsilon] > 0$. Clearly, for the ethnic minority the welfare in this scenario is $-\psi$.

Notice, overall welfare (in the sense of population-weighted average of the welfare of the different ethnic groups) when ethnic diversity is low ($\lambda > \hat{\lambda}$) is given by

$$1 + \lambda E[\epsilon] - (1 - \lambda)\psi.$$ 

Now, this term could be higher/equal to/lower than unity depending upon the other parameter values. In particular, if the direct costs to the minorities from discrimination are sufficiently high, then overall welfare here would be lower. In terms of the distribution of the welfare, clearly there is more inequality as compared to the case of $\lambda < \hat{\lambda}$.

Now we turn to the dictatorship regime. For $\lambda < \hat{\lambda}$, only the $E$ good is provided. But notice $\mu^* > 0$ and hence it is possible that $U_\lambda < 1$ (compare with the corresponding scenario under democracy). Moreover, the level of welfare for any member of the $\lambda$--group is increasing in diversity (follows from Observation 5).

For the $\lambda > \hat{\lambda}$ case, we have only the $G$ good being provided. Once again $\mu^* > 0$ and hence overall welfare is clearly below 1 (compare with the corresponding scenario under democracy). Moreover, the level of welfare is decreasing in diversity (follows from Observation 8). Notice, the minority group is better off here than under democracy in spite of $\mu^* > 0$.

5 Discussion

Here we discuss some historical events which when analysed within the framework of our model appear to corroborate the model’s predictions. In each of the following cases, there is a change in regime (from democracy to dictatorship or vice versa) which allows us to make a comparison in terms of changes in public spending or broadly speaking, public policy. The implicit assumption is that the level of ethnic diversity for a country is largely unaffected by the regime change. We consider countries with both high and low ethnic diversity to illustrate the workings of our theory.

**Rwanda:** The Hutu community is the major ethnic group in Rwanda (over 80%) and a dominant minority group is the Tutsi (about 15%). The genocide involving the massacre

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28It is not possible to provide a sharper comparison without assuming some specific functional form for the distribution $F$. This is required in order to solve for $\mu^*$ explicitly.
of Tutsi (and moderate Hutu) civilians by certain armed groups of Hutus which happened in 1994 has received much attention. This started soon after the fatal plane crash of the Rwandan president, Juvenal Habyarimana. Incidentally, Habyarimana (a Hutu from the northern part of Rwanda) actually staged a military coup to seize power from the previous president, Gregoire Kayibanda (a Hutu political leader) in 1973. Kayibanda’s tenure, which was a democracy (although far from perfect) was characterised by several incidents of Tutsi persecution. As Prunier (1995) notes:

“When he [Juvenal Habyarimana] took power in a bloodless coup on 5 July 1973, there was widespread popular relief, even among the Tutsi whose security the new regime immediately guaranteed... All in all, life was difficult for the Tutsi who were victims of institutional discrimination, but in everyday life it was quite tolerable. Compared to the Kayibanda years, things had improved, even to the point where some well-known Tutsi businessmen had made fortunes and were on very good terms with the regime. The unspoken understanding was ‘Do not mess around with politics, this is a Hutu preserve’. As long as Tutsi stuck to that principle, they were generally left in peace.”

By most accounts, one infers that until 1990 only low-level incidents of violence against the minority Tutsi had occurred under Habyarimana’s rule — nothing on the same scale as the persecution and mass killings that periodically took place before the 1973 coup. In the context of our model, the Rwandan situation describes a society which is fairly homogeneous in ethnicity and which sees a transition from democracy to dictatorship. By Propositions 1 and 2, such a society would be characterised by neglect of minorities under democracy and (relatively) equal treatment under dictatorship. The above account seems consistent with such predictions.

The political situation in post-genocide Rwanda has been described as a quasi-autocracy, given President Kagame’s influence (see Blouin and Mukand (2018)). In terms of our theory, this ought to be a period characterised by equal treatment of the ethnic groups. Blouin and Mukand (2018) exploit variation in exposure to the government’s radio propaganda due to the mountainous topography of Rwanda. Results of their lab-in-the-field experiments demonstrate that individuals exposed to government propaganda have lower salience of ethnicity, increased inter-ethnic trust and show more willingness to interact face-to-face with members of another ethnic group. Therefore, this government is improving relations between ethnic groups rather than engage in discrimination, which is consistent with our theory.

Indonesia: Indonesia is an ethnically diverse country where largest ethnic group is the Javanese, who comprise about 42% the population and are politically and culturally dominant (see e.g., Kingsbury (2003)). President Suharto’s regime (also called the “New Order”) lasted over 30 years (1966–1998) and can be safely classified as a non-democracy for most of its duration. In May 1998, there was a massive unrest in the country. In particular, there was

\(^{29}\)Habyarimana created the Mouvement Revolutionnaire National pour le Developpement (MRND) as the country’s only legal party in 1975.
targeted violence against a minority group, specifically, the Chinese-Indonesians in several parts of the country (see Panggabean and Smith (2011)). Notice, this is akin to favouring the dominant ethnic group at the expense of the minorities in a setup characterised by high ethnic diversity under a dictatorship (providing $E$ rather than $G$ in terms of our model as in Proposition 2).

This pattern of persecution was abandoned — at least to an extent — after the transition to democracy in the 21st century. Tan (2005) observes: “Since the end of the repressive Suharto regime, aside from some localized incidents, the ethnic Chinese have been left more or less alone.” After the fall of Suharto, numerous discriminative laws were recalled and others promoting unity were passed.\(^{30}\) This is again in line with our result in Proposition 1 which states that democracies with high ethnic diversity do not discriminate in terms of public spending across ethnic groups (i.e., they provide $G$ rather than $E$).

**Myanmar:** The Rohingya community in Myanmar have recently been in the news for the treatment they have been receiving at the hands of government forces. Lindblom et al. (2015) state:

“The Rohingya Muslims in Myanmar’s Rakhine State have suffered serious and persistent human rights abuses. Myanmar authorities, security forces, police, and local Rakhine actors have engaged in widespread violence, acts of torture, arbitrary detention, rape, and other crimes causing serious physical and mental harm. The scale of these atrocities has increased precipitously since 2012.”

Incidentally, Myanmar saw a full-scale transition to democracy in 2015 where Aung San Suu Kyi’s party won a landslide victory, taking 86% of the seats in the Assembly of the Union. Although she was prohibited from becoming the President due to a clause in the constitution she assumed the newly created role of State Counsellor, making her the *de facto* head of government.

Aung San Suu Kyi has come under sharp criticism for her perceived indifference to the plight of the Rohingya community. In a 2015 BBC News article, it was suggested that Suu Kyi’s silence over the Rohingya issue is due to a need to obtain support from the majority Bamar ethnicity as she is in “the middle of a general election campaign”.\(^{31}\) Given that the Bamar ethnic group constitutes about 68% of the Burmese population (while the rest is composed of several small ethnicities), Myanmar fits the description of being a less ethnically diverse society in terms of our model. By Propositions 1 and 2, such a society would demonstrate neglect of minorities under democracy and (relatively) equal treatment under dictatorship. To be sure, the Rohingya community has been suffering under the military government too,

\(^{30}\)President Habibie passed legislation requiring the elimination of the terms pribumi and non-pribumi (native Indonesian and non-native) in 1998. In 2002, Chinese New Year was declared a national holiday. However, some discriminative legislation still remains. Chinese Indonesians have been “embraced” by the government, with numerous mixed-ethnic cultural presentations and media activity. By 2004, there were three Chinese Indonesian members of the Peoples Representative Council, as well as one cabinet member.

\(^{31}\)The BBC article can be accessed at http://www.bbc.co.uk/news/world-asia-32974061
but as noted above, there has been an uptick in the violence against them in the recent “democracy” years. All of this is fairly consistent with the predictions of our theory.

6 Conclusion

The issue of discrimination against minorities is of significant interest to researchers and policy-makers alike. Given the recent interest in the role of institutions on the workings on the economy, it is natural to ask if “superior” institutions like democracy can automatically alleviate such discrimination. Here we have attempted to take on this question with the help of a simple model. We have tried to capture discrimination in public policy in a stylised way by allowing the government (popularly elected or otherwise) to either engage in public spending on genuinely public goods or on providing what only members of the dominant ethnic group benefit from — the latter option exemplifies discrimination. We next analyse this model under two starkly different political regimes — namely, democracy and dictatorship.

Our model brings out the contrast in public spending patterns — specifically, discriminatory spending — by highlighting the tensions that drive behaviour under democracy and dictatorships. A society with a relatively small dominant community (and hence largely diverse) is likely to see a more homogeneous pattern of public spending under democracy as compared to one where the dominant community is a sizeable super-majority. In the latter case, targeting the dominant community is enough to guarantee electoral success. However, the considerations are altogether different under a dictatorship where the dictator has to think of pre-empting any revolution which is undertaken in the hope of moving to democracy. Here different ethnic groups would have different motives based on what they expect under democracy.

Our theory is capable of interpreting certain historical events involving regime changes. Our model predicts that such a change would either favour or dis-favour minorities depending upon the size of the dominant ethnic group. We discuss a few instances — namely, Rwanda, Indonesia and Myanmar — in the framework of our model and observe that our predictions are consistent with these cases.

Coming back to the question regarding democracy mitigating concerns regarding discrimination, our position is not without sufficient scepticism. As our analysis demonstrates, minorities may well face less discrimination under dictatorships. That is not to say that democracy necessarily imposes the ”tyranny of the masses” — we outline conditions in our model as to when they do not. In conclusion, our findings suggest that extra safeguards (reservation of posts, quotas, etc.) need to be in place so as to rescue minorities from unfair treatment in illiberal democracies.
Appendix

Proof. [Observation 1.] Start with \((e_A = e_B = 0, g_A = g_B = 1)\). Here, each party gets an expected payoff of \(1/2\). Suppose party \(A\) deviates to \(\tilde{e}_A > 0\). This implies that \(A\) will definitely lose the votes from the \((1 - \lambda)\)-group (since they get a payoff of 1 from party \(B\) and \(A\) cannot guarantee them 1 if \(\tilde{e}_A > 0\)). Hence, the optimal deviation for \(A\) involves \(\tilde{e}_A = 1/\lambda\).

Now, a voter \(i\) of the \(\lambda\)-group votes for \(A\) if

\[ \tilde{e}_A + \epsilon_i > g_B. \]

Otherwise, voter \(i\) favours \(B\) and this happens with probability \(F(g_B - \tilde{e}_A) = F(1 - 1/\lambda)\).

In other words, he votes for \(A\) with probability \(1 - F((\lambda - 1)/\lambda)\). Hence expected votes for \(A\) is given by

\[ V(\lambda) \equiv \lambda \{ 1 - F((\lambda - 1)/\lambda) \}. \]

Now, \(V(\lambda) < 1/2\) for \(\lambda = 1/2\) since \(f > 0\) everywhere on the real line. This implies that the deviation by \(A\) is unprofitable for \(\lambda = 1/2\). By the continuity of \(V(.)\) in \(\lambda\), this implies the existence of some \(\delta\)-neighborhood around \(\lambda = 1/2\) such that \(V < 1/2\) in that \(\delta\)-neighborhood. This gives a \(\lambda > 1/2\) such that \(V(\lambda) < 1/2\) for every \(\lambda \in [1/2, \lambda]\) making \((e_A = e_B = 0, g_A = g_B = 1)\) an equilibrium in that \(\lambda\)-interval.

For uniqueness, note the following. In any equilibrium, each party must have an expected payoff of \(1/2\) otherwise ‘mimicry’ is always a profitable deviation. Any equilibrium apart from \((e_A = e_B = 0, g_A = g_B = 1)\) necessarily involves at least one party offering a positive amount of \(E\). The arguments above establish that any such platform must necessarily yield a payoff lower than \(1/2\) when the other party proposes to spend the entire budget on \(G\). Thus, \((e_A = e_B = 0, g_A = g_B = 1)\) is the only equilibrium in that \(\lambda\)-interval. \(\blacksquare\)

Proof. [Observation 2.] Start with \((e_A = e_B = 1/\lambda, g_A = g_B = 0)\). Here, each party gets an expected payoff of \(1/2\).

Suppose party \(A\) deviates to \((g'_A, e'_A) > 0\). This implies that \(A\) will definitely lose the votes from the \(\lambda\)-group. To see why, note the following.

For any \(i\) in the \(\lambda\)-group, the payoff from \(B\) is \(1/\lambda + \epsilon_i\). From \((g'_A, e'_A)\), the same voter’s payoff is \((1 - g'_A)/\lambda + g'_A + \epsilon_i\). But

\[ (1 - g'_A)/\lambda + g'_A + \epsilon_i = 1/\lambda + g'_A(1 - 1/\lambda) + \epsilon_i < 1/\lambda + \epsilon_i \]

since \(1/2 \leq \lambda < 1\). Hence, \((g'_A, e'_A) \geq 0\) cannot be a profitable deviation for \(A\) for any \(\lambda \in [1/2, 1]\).

Now suppose \(A\) deviates to \(g'_A = 1\). Recall \(V(\lambda) \equiv \lambda \{ 1 - F((\lambda - 1)/\lambda) \}\) from Observation
1. Note that $V(\lambda)$ here is the payoff to $B$ when $A$ proposes $g'_{A} = 1$ and $B$ proposes $(e_{B} = 1/\lambda, g_{B} = 0)$ for any given $\lambda$. Hence, $V(\lambda) \geq 1/2$ implies that $A$’s deviation is not profitable. Now, $V(\lambda) > 1/2$ for $\lambda = 1$ since we have $F(0) < 1/2$. By the continuity of $V(.)$ in $\lambda$, this implies the existence of some $\delta'$-neighborhood around $\lambda = 1$ such that $V > 1/2$ in that $\delta'$-neighborhood. This gives a $\lambda > 1/2$ such that $V(\lambda) > 1/2$ for every $\lambda \in [\lambda, 1)$ making $(e_{A} = e_{B} = 1/\lambda, g_{A} = g_{B} = 0)$ an equilibrium in that $\lambda$-interval.

For uniqueness, note the following. Any equilibrium apart from $(e_{A} = e_{B} = 1/\lambda, g_{A} = g_{B} = 0)$ necessarily involves at least one party offering a positive amount of $G$. The arguments above establish that any such platform must necessarily yield a payoff lower than $1/2$ when the other party proposes to spend the entire budget on $E$. Thus, $(e_{A} = e_{B} = 1/\lambda, g_{A} = g_{B} = 0)$ is the only equilibrium in that $\lambda$-interval. 

**Proof.** [Observation 3.] By inspection it is clear that the derivative of $V(\lambda)$ w.r.t. $\lambda$ — call it $V'_{\lambda}$ — is of ambiguous sign. As noted earlier, $V(\lambda) < 1/2$ for $\lambda = 1/2$ and $V(\lambda) > 1/2$ for $\lambda = 1$. Hence, by the continuity of $V$ in $\lambda$, there exists $\lambda \in [1/2, 1)$ such that $V(\lambda) = 1/2$. Let $\bar{\lambda}$ be such a $\lambda$. We will argue that there can only be one such $\lambda$.

Observe the following equation:

$$\lambda\{1 - F((\lambda - 1)/\lambda)\} = 1/2.$$

Let $x \equiv 1/\lambda$. So, $x \in (1, 2]$. Hence, the above equation can be written as

$$x + 2F(1 - x) = 2.$$

If we can show that the solution to the above equation is unique, we are done. Define $y(x) \equiv x + 2F(1 - x)$. Note that

$$\frac{\delta y}{\delta x} = 1 - 2f'(1 - x).$$

Also,

$$\frac{\delta^2 y}{\delta x^2} = 2f''(1 - x).$$

Given that $x > 1$ and $f'(z) > 0$ whenever $z < 0$ (by the unimodality and symmetry of $f$ around $\mu_{r}$), this implies $\frac{\delta^2 y}{\delta x^2} > 0$. Hence, $\frac{\delta y}{\delta x}$ is increasing in $x$ for $x > 1$.

Note that $y(2) > 2 > y(1)$. Hence, $\frac{\delta y}{\delta x}$ must be positive for some $x \in (1, 2]$. Combining this with the fact that $\frac{\delta y}{\delta x}$ is increasing in $x$ for $x > 1$, we get that there is a unique $x$ (and hence a unique $\lambda = 1/x$) where $y(x) = 2$. This completes the proof. 

**Proof.** [Observation 4.] Both parties offering $g = 1$ is an equilibrium. The best possible unilateral deviation is $e = 1/\bar{\lambda}$ which yields a payoff of $1/2$ given that $V(\bar{\lambda}) = 1/2$. Both
parties offering \( e = 1/\lambda \) is also an equilibrium. The best possible unilateral deviation is \( g = 1 \) which yields a payoff of \( 1/2 \) given that \( V(\lambda) = 1/2 \). Also, \((e_A = 1/\lambda, g_B = 1)\) and \((g_A = 1, e_B = 1/\lambda)\) are also equilibria platforms arising from the fact that \( V(\lambda) = 1/2 \). Finally, each party mixing between \( g = 1 \) and \( e = 1/\lambda \) with any positive probability on one of the pure strategies will constitute an equilibrium. This completes the proof.

**Proof.** [Observation 5.] The first part, i.e., \( e_D^* > 0 \) and \( g_D = 0 \) follows immediately from the standard Lagrangean FOCs. So the optimal \( e_D \) is either \( 1 - \mu/\lambda \) or lies in \((1 - \mu/\lambda, 1/\lambda)\).

If the optimal \( e_D \) is \( 1 - \mu/\lambda \), then the second part is obvious. For an interior \( e_D^* \), to see why the second part holds, recall assumption A1 and note that the FOC w.r.t. \( e_D \) can be written as:

\[
\frac{f(1 - e_D)}{1 - F(1 - e_D)} = \frac{\lambda}{1 - \lambda e_D}.
\]

By A1, the LHS of the above is (weakly) decreasing in \( e_D \).

Now suppose \( \frac{\partial e_D^*}{\partial \lambda} > 0 \). Consider an increase in \( \lambda \). This immediately implies that the RHS increases. But if \( \frac{\partial e_D^*}{\partial \lambda} > 0 \), then the LHS must decrease by A1. Therefore, \( \frac{\partial e_D^*}{\partial \lambda} \leq 0 \).

Now suppose \( \frac{\partial e_D^*}{\partial \lambda} = 0 \). Consider an increase in \( \lambda \). Again, the RHS decreases. Now the LHS is unchanged leading to a contradiction.

Hence, \( \frac{\partial e_D^*}{\partial \lambda} < 0 \) is the only possibility. ■

**Proof.** [Observation 6.] Suppose \( D \) chooses to offer a positive amount of \( E \). The members of the minority group will not revolt. But all the members of the \( \lambda \)-group will revolt (since \( 1/\lambda > g_D + e_D \)). This implies that the probability that \( D \) is removed is \( p = \lambda \). Hence, \( D \)'s objective function is \((1 - \lambda)\mu \). So \( D \) will choose to steal the maximum amount permissible, i.e., \( \mu \).

However, \( D \) can improve on this by setting \( g_D = (1 - \mu) \) and \( e_D = 0 \). This guarantees \( D \) a payoff of \( \mu \) when the revolt fails. Moreover, the success probability of the revolt is now strictly below \( \lambda \) since some members of the \( \lambda \)-group (who have \( 1/\lambda + \epsilon_i < (1 - \mu) \)) will choose not to revolt.

Hence, \( D \) will set \( e_D^* = 0 \) and \( g_D^* > 0 \) ■

**Proof.** [Observation 7.] For some \( \tilde{\lambda} \in (\hat{\lambda}, 1) \), \( D \)'s optimal choice of \( \mu \) for diversity level \( \tilde{\lambda} \) is in \((0, \mu)\). Given that the optimal choice \( \mu^* \) uniquely solves \( \theta(\mu^*, \tilde{\lambda}) = 0 \), this implies \( \theta(\mu^*, \tilde{\lambda}) < 0 \).

Re-write \( \theta(\mu, \lambda) \) as

\[
\lambda[1 - F(1 - \mu - (1/\lambda)))]\left(\frac{1}{\lambda[1 - F(1 - \mu - (1/\lambda))]} - 1 - \frac{\mu f(1 - \mu - (1/\lambda))}{1 - F(1 - \mu - (1/\lambda))}\right).
\]
So the sign of \( \theta(\mu, \lambda) \) is the same as that of \( \left( \frac{1}{\lambda[1-F(1-\mu-(1/\lambda))] - 1 - \frac{\mu f(1-\mu-(1/\lambda))}{1-F(1-\mu-(1/\lambda))} \right) \).

Hence, \( \left( \frac{1}{\lambda[1-F(1-\mu-(1/\lambda))] - 1 - \frac{\mu f(1-\mu-(1/\lambda))}{1-F(1-\mu-(1/\lambda))} \right) < 0 \) for \( \mu = \overline{\mu} \) and \( \lambda = \hat{\lambda} \).

Now starting with \( \mu = \overline{\mu} \) and \( \lambda = \hat{\lambda} \), consider an increase in \( \lambda \). This implies \( (1 - \overline{\mu} - (1/\lambda)) \) increases. Hence, by A1, \( \frac{\mu f(1-\mu-(1/\lambda))}{1-F(1-\mu-(1/\lambda))} \) weakly increases. Therefore, if we can show that \( \lambda[1-F(1-\mu-(1/\lambda))] \) weakly increases in \( \lambda \), then we have \( \left( \frac{1}{\lambda[1-F(1-\mu-(1/\lambda))] - 1 - \frac{\mu f(1-\mu-(1/\lambda))}{1-F(1-\mu-(1/\lambda))} \right) < 0 \) for \( \mu = \overline{\mu} \) and for every \( \lambda \in (\hat{\lambda}, 1) \).

Differentiation of \( \lambda[1-F(1-\mu-(1/\lambda))] \) w.r.t. \( \lambda \) yields the following:

\[
1 - F(1-\mu-(1/\lambda)) - \frac{1}{\lambda} f(1-\mu-(1/\lambda)).
\]

By A3, \( f(0) \leq 1/4 \) and \( f(1-\mu-(1/\lambda)) < f(0) \) since \( (1 - \mu - (1/\lambda)) \) < 0. Hence, \( \frac{1}{\lambda} f(1-\mu-(1/\lambda)) < 1/2 \). Moreover, \( F(1-\mu-(1/\lambda)) < F(0) < 1/2 \).

Hence, \( 1 - F(1-\mu-(1/\lambda)) - \frac{1}{\lambda} f(1-\mu-(1/\lambda)) > 0 \).

Thus, we have \( \left( \frac{1}{\lambda[1-F(1-\mu-(1/\lambda))] - 1 - \frac{\mu f(1-\mu-(1/\lambda))}{1-F(1-\mu-(1/\lambda))} \right) < 0 \) for \( \mu = \overline{\mu} \) and for every \( \lambda \in (\hat{\lambda}, 1) \).

Hence, \( \theta(\mu, \lambda) < 0 \) for \( \mu = \overline{\mu} \) and for every \( \lambda \in (\hat{\lambda}, 1) \). This, in turn, implies that \( D \)'s optimal choice of \( \mu \) for every \( \lambda \in (\hat{\lambda}, 1) \) lies in \((0, \overline{\mu})\) which completes the proof. 

**Proof.** [Observation 8.] Part (i) obtains when \( \theta(\mu, \lambda) \geq 0 \) for \( \mu = \overline{\mu} \) and for every \( \lambda \in (\hat{\lambda}, 1) \).

For part (ii), if there is some \( \hat{\lambda} \in (\hat{\lambda}, 1) \) such that \( \theta(\overline{\mu}, \hat{\lambda}) < 0 \) then the (interior) solution \( \mu^* \) to the FOC satisfies the following

\[
1 - F(1 - \mu^* -(1/\lambda)) + \mu^* f(1 - \mu^* -(1/\lambda)) = 1/\lambda
\]

for \( \lambda = \hat{\lambda} \). By Observation 7, the optimal \( \mu \) must lie in the interior and hence satisfy the FOC for every \( \lambda \geq \hat{\lambda} \). Therefore, for the above FOC equation, the total derivative of the LHS w.r.t \( \lambda \) should equal that of the RHS for every \( \lambda \geq \hat{\lambda} \).

Note, the derivative of the RHS w.r.t \( \lambda \) is \(-1/\lambda^2\). Differentiating the LHS w.r.t. \( \lambda \) yields

\[
f(1-\mu^*-(1/\lambda)) \left( \frac{\partial \mu^*}{\partial \lambda} - \frac{1}{\lambda^2} \right) + \frac{\partial \mu^*}{\partial \lambda} \left[ f(1-\mu^*-(1/\lambda)) - \mu^* f'(1-\mu^*-(1/\lambda)) \right] + \frac{\mu^*}{\lambda^2} f'(1-\mu^*-(1/\lambda))
\]

which after re-arranging terms can be written as

\[
\left( \frac{\partial \mu^*}{\partial \lambda} - \frac{1}{\lambda^2} \right) \left[ f(1-\mu^*-(1/\lambda)) - \mu^* f'(1-\mu^*-(1/\lambda)) \right] + \frac{\partial \mu^*}{\partial \lambda} \cdot f(1 - \mu^* -(1/\lambda)).
\]

Call this expression \( \tau(\lambda) \). So \( \tau(\lambda) = -1/\lambda^2 \) for every \( \lambda > \hat{\lambda} \).
Note \((1 - \mu^* - 1/\lambda)) < 0\) (since \(\mu^* \geq 0\) and \(\lambda < 1\)). Also, \(f'(x) > 0\) for all \(x \leq 0\). Under assumption A3 this leads to \(0 < f(1 - \mu^* - 1/\lambda)) < 1\).

Also, \(f(1 - \mu^* - (1/\lambda)) > \mu^* f'(1 - \mu^* - (1/\lambda))\) by A2 which implies \[
\left[ f(1 - \mu^* - (1/\lambda)) - \mu^* f'(1 - \mu^* - (1/\lambda)) \right] > 0.
\]

Now, suppose \(\partial \mu^*/\partial \lambda > 0\). If in fact, \(\partial \mu^*/\partial \lambda \geq 1/\lambda^2\), then \(\tau(\lambda)\) clearly exceeds 0 leading to a contradiction.

Next consider \(\partial \mu^*/\partial \lambda \in (0, 1/\lambda^2)\). In this case
\[
\tau(\lambda) > \left( \frac{\partial \mu^*}{\partial \lambda} - \frac{1}{\lambda^2} \right) \left[ f(1 - \mu^* - (1/\lambda)) \right] + \frac{\partial \mu^*}{\partial \lambda} \cdot f(1 - \mu^* - (1/\lambda))
\]
\[
> \left( \frac{\partial \mu^*}{\partial \lambda} - \frac{1}{\lambda^2} \right) \left[ f(1 - \mu^* - (1/\lambda)) \right] > \left( - \frac{1}{\lambda^2} \right) \left[ f(1 - \mu^* - (1/\lambda)) \right] > -\frac{1}{\lambda^2}
\]
where the first inequality follows from \(f'(1 - \mu^* - (1/\lambda)) > 0\) and the last inequality follows from \(0 < f(1 - \mu^* - (1/\lambda)) < 1\). Hence, it cannot be that \(\partial \mu^*/\partial \lambda > 0\).

Suppose \(\partial \mu^*/\partial \lambda = 0\). Then
\[
\tau(\lambda) = \left( - \frac{1}{\lambda^2} \right) \left[ f(1 - \mu^* - (1/\lambda)) - \mu^* f'(1 - \mu^* - (1/\lambda)) \right]
\]
\[
> \left( - \frac{1}{\lambda^2} \right) \left[ f(1 - \mu^* - (1/\lambda)) \right] > -\frac{1}{\lambda^2},
\]
thus leading to a contradiction. This establishes \(\partial \mu^*/\partial \lambda < 0\) for every \(\lambda > \tilde{\lambda}\).
References


