Advancing at the Cost of Others? The impact of an Education Intervention on Performance Inequality among Primary School Learners

DRAFT

Volker Schöer
Adeola Oyenubi

University of the Witwatersrand

Abstract: The crisis in the South African education sector is a major constraint to any meaningful transformation of the South African society. In response, the government has increased its efforts to address obstacles to learning. One potentially promising initiative has been an instructional change intervention aimed at improving pedagogical practices of foundation phase teachers. This intervention has been tested through a number of experimental studies that have consistently shown a positive difference in the mean performance of learners in intervention schools compared to learners in control schools. However, following the critique by Deaton and Cartwright (2018), simply comparing means could miss important nuances. For instance, a number of these studies have shown that only higher performing learners benefit from this intervention while lower performing learners potentially do worse. We investigate the impact of the instructional change intervention on the relative performance of learners within the treatment arms. Specifically, we unpack the impact of the intervention on the inequality in learning outcomes by testing if changes in inequality measures are similar to the change observed at the mean between the treatment arms. Using a range of inequality measures, we show that while there is natural polarization of learner performance in the control schools, the intervention exacerbates the polarization and increases performance inequality among learners in treatment schools.

JEL classifications: I24, I25, C18, C90
Introduction

Since the late 1990s, Randomized Controlled Trials (RCTs) have increasingly become the preferred method of assessing education interventions. This shift has been mainly driven by the need to provide robust evidence that establishes whether an intervention is having a discernible and measurable effect on learners’ outcomes (Torgerson and Torgerson 2001).

The attractiveness of the RCT method is its simplicity. The randomization allows to focus entirely on the intervention effect as all other factors are assumed to be equally balanced across the treatment and the control groups due to random assignment. As we cannot observe the counterfactual for the treatment group, we simply use the average of the control group to replace the missing counterfactual and calculate the average treatment effect as the difference in the means.

However, one central assumption of an RCT is that the difference in two group means is also the mean of the individual differences, i.e. the treatment effect. As noted by Deaton and Cartwright (2016) this is a limitation since a similar argument does not apply to other functionals of the distributions being compared, for instance this might not necessarily be true for the median. However, many interesting questions involving the political economy of programmes involve knowledge of the distribution of impacts, or features of it (Heckman et al., 1997). Heckman et al. (1997) noted that the case for mean impact rests on two key assumptions (a) increase in total output increases welfare and (b) the undesirable distributional aspects of programmes are either unimportant or are offset by transfers. These assumptions are strong because many programmes produce outcomes that cannot be redistributed (for instance, outcomes of educational programmes).

The key problem with making inference on other distributional functionals for an RCT is that we cannot simultaneously observe the same person under the two treatment regimes. Therefore, it is not possible to determine programme impact for individuals. Hence the distribution of impacts and its features are unknown except if one is willing to make certain assumptions. For example, one can assume that the impact is constant across beneficiaries,
this assumption will mean that the distribution of impact is degenerate and the mean, median and all other quantiles are identical\textsuperscript{1}.

However, Firpo (2005) as well as Deaton and Cartwright (2018) noted that a policymaker may be interested in parameters that capture summarized distributional effect of treatment. For example, the effect of treatment on the dispersion of outcome. This may be captured by inequality measures such as the variance, the Gini coefficient, the quantile ratio and other inequality measures.

The distribution of educational outcomes is one example. For educational interventions, it is important that the intervention should be pitched at the right level for it to be effective (Banerjee et al., 2015; Kaffenberger & Pritchett, 2019). Failure to do this may result in a situation where the programme benefits only certain learners at the expense of others thereby inducing inequality in the distribution of outcome (with or without a mean effect). This can lead to the “Matthew effect” whereby gains for participating in an educational intervention can be higher for the strongest learners relative to other students for whom the intervention is pitched at a level beyond their learning ability (Stanovich, 1986)

We investigate the possibility of a Matthew effect as an outcome of an instructional change intervention in South African primary schools. Fleisch et al, (2017) report descriptive evidence of a Matthew effect when analysing the impact of the Reading Catch-up Programme (RCUP). The existence of a Matthew effect suggests that inequality in the achievement distribution of the treatment group may have increased relative to the control group. This suggests that weaker students may have been better off in the control group. This is particularly problematic as the short-term intervention might have disadvantaged learners even further.

Following Stanovich (1986), early deficits in learning to read are likely to exacerbate the achievement gap in later years.

We therefore re-analyse the RCUP data with a focus on the effect of the programme on the inequality in achievement relative to learners in the control group who have undergone the regular curriculum. This inequality may come in the form of stagnation or retardation in the performance of some treatment group members relative to control group students who study

\textsuperscript{1} Features like quantile treatment effect for example are only identifiable if the assumption of rank invariance is invoked. Like the constant effect assumption this assumption too is too strong.
under the regular curriculum. Since inequality is based on the overall distribution of outcomes (as against quantiles which are based on individual outcomes) assumption of rank invariance might not be necessary to identify this functional. This is because popular inequality measures satisfy the principle of anonymity. Anonymity implies that inequality measures should not change if individuals are relabelled which suggests that differences in inequality measures across treatment arms might coincide with the effect of the programme on inequality even when rank invariance is not reasonable (Firpo, 2005).

Reading Catch-Up Program: an Instructional Change intervention

The Reading Catch-Up Program (RCUP) was designed to improve literacy competencies of Grade 4 primary school learners from under-resourced schools. The focus of the intervention is to change the instructional practice of the teacher to align the teaching with current best practice of teaching literacy and the South African curriculum.

It includes a set of components that are aimed at changing the instructional practice of the primary school teacher (See Fleisch et al 2017 for detailed description). XXX primary schools in the Pinetown area close to Durban were selected with a total of XXX learners.

12 weeks with a pre- and post-test instrument that tested spelling, language, comprehension and writing competencies of these Grade 4 learners.

The research team found that the intervention did not have a significant effect on the overall literacy competencies of learners in treatment schools but had some effect on lower literacy competencies including spelling and language.

Methods

To investigate if the RCUP programme has an effect on inequality in educational achievement we use a battery of methods that is capable of highlighting the effect of the programme on inequality measures.

Our first set of analysis uses the Recentered Influence Function (RIF) methodology (Firpo, 2005; Firpo, Fortin and Lemieux, 2009). Following Firpo (2005) we define inequality treatment effect on the treated as the difference in outcome inequality between the treated and the
counterfactual. Unlike Firpo (2005) the ignorability and common support assumptions are automatically satisfied since we are using randomized data. This method allows us to estimate the impact of the programme on other distributional functionals like variance, quantiles and various measure of inequality using RIF regressions.

The result in Fleisch et al, (2017) show conditional quantile results as a way to show the existence of Matthew effect in the data. We re-estimate the quantile effect using RIF regression. Unlike the result in Fleisch et al, (2017) this is an unconditional quantile effect. We then proceed by estimating the effect of the programme on the variance, Gini coefficient and Generalized entropy measures of inequality in the outcome distribution across treatment arms. For the theoretical underpinnings of this method see Firpo (2005).

Our second and main analysis dig dipper into what happens to individual learners within and across the treatment arms. We use relative distribution methods which is a statistical tool for fully representing differences in distributions. Developed by Handcock and Morris (1999), this method can be used to decompose changes in distributions into location and shape components and identify the nature and direction of movement in mass in the distributions being compared. Originally developed to investigate changes in earning distributions (see Handcock and Morris (1999)), these measure has been used in health economics to track the changes in BMI (see Contoyannis and Wildman (2007)).

In this study, we adopt this methodology to investigate the changes in distribution as a consequence of treatment. In this context, we can answer questions such as; Relative to the control distribution is the change in treatment distribution driven by a location change or a shape change? Furthermore, one can investigate changes in the treatment distribution relative to its state before treatment. This questions will help unveil the nature of differences that occur in the distribution.

More formally, let $Y_0$ be the distribution of outcome in the control group, designated the reference group. The same outcome denoted $Y_1$ is observed for the treatment population. Let $F_0(y_0)$ and $F_1(y_1)$ be the cumulative distribution function CDF of $Y_0$ and $Y_1$. Relative distribution can be defined as the distribution of a random variable $R$, where $R = F_0(y_1)$. $R$

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2 This is the first time this methodology is being used in this context to the best of our knowledge.

3 Handcock and Morris (1999) called this the comparison group but we refrain from this terminology so that it will not be misconstrued for the control group.
then indicates the percentile rank of \( Y_1 \) if it were placed in the distribution of \( Y_0 \). In our context, a point on the relative CDF is given by the proportion of treatment distribution that falls below the \( r^{th} \) percentile point in the control distribution consistent with a given value of \( Y_1 \) (Contoyannis and Wildman, 2007). In the case of all values of \( Y_0 \) we can write the relative CDF or the CDF of \( R \) as

\[
G(r) = F_1(F_0^{-1}(r)) = F_1(Q_0(r)), \quad 0 \leq r \leq 1
\]

where \( Q_0(r) \) is the quantile function of \( F_0 \) and \( r \) is a point on the CDF of \( F_0 \). The PDF of \( R \), the relative density is given by the derivative of \( G(r) \) with respect to \( r \)

\[
g(r) = \frac{f_1(Q_0(r))}{f_0(Q_0(r))}
\]

The relative density can, therefore, be interpreted as a density ratio\(^4\). Note that it is also a proper PDF as it integrates to 1 over the unit interval. If the two distributions are identical, then the relative density is a uniform distribution on \([0,1]\). We use this approach to decompose the difference between the control (the reference group) and the treatment distributions into changes in location and changes in shape. Note that since our data is from an RCT, it sufficient to use the outcome data only since all the other covariates are balanced i.e. the differences in outcome can only be explained by the treatment states.

In terms of decomposition, if the treatment has the sole effect of shifting (either additively or multiplicatively) the distribution of outcome for treated observations, then only a location change will be observed\(^5\). If this is not the case then any residual difference that remains after location adjustment is due to changes in shape which include scale, skew and other distributional characteristics like different forms of inequality. Shape differences can manifest in form of a tendency toward bimodality or denser tails, signifying that while some students benefit others actually performed worse relative to what their performance would have been under the control state.

\(^4\) To see this more clearly one can express \( g(r) \) explicitly in terms of the original measurement scale \( y \). Let the \( r^{th} \) quantile of \( R \) be denoted by \( y_r \) on the original measurement scale. The PDF is then

\[
g(r) = \frac{f(y_0)}{f_0(y_r)} y_r = Q_0(r) \geq 0
\]

see Handcock and Morris (1999) for details and examples.

\(^5\) This require that the treatment distribution \( F_1(y_1) = F_0(y_0 + c) \) i.e. a pure location shift.
Decomposition into location and shape effect proceeds by first generating the counterfactual distribution $f_{0L}(y_r)$ (this is not to be confused with the traditional meaning of counterfactual distribution in the treatment effect framework which is represented by the control group), by counterfactual in this context we mean a distribution that has the same location (mean or median depending on how one chose to measure location) as the treatment distribution $f_1(y_1)$ but the same shape as the control distribution $f_0(y_0)$. This is achieved by adding the difference between the location measure (note that when the location measure is the mean this will be the average treatment effect) to $f_0(y_0)$. A relative distribution constructed as $\frac{f_{0L}(y_r)}{f_0(y_r)}$ will yield a density ratio which will be uniformly distributed if the treatment effect is zero (since $f_{0L}(y_r) = f_0(y_r)$) i.e. no location difference between the treatment and control group. The shape element of the decomposition is the residual difference which is given by $\frac{f_1(y_1)}{f_{0L}(y_r)}$, a departure from uniform distribution for this relative density indicate differences in distributional characteristic other than location between the treatment and the control group. Note that the decomposition can be written as:

$$\frac{f_1(y_1)}{f_0(y_0)} = \frac{f_{0L}(y_r)}{f_0(y_r)} \times \frac{f_1(y_1)}{f_{0L}(y_r)}$$

The density ratio for the location effect is a proper density (i.e. it integrates to 1). On the other hand, the density ratio for the shape effect is generally not a proper density. This is because $f_{0L}(y_r)$ is used in the denominator of the shape effect to ensure comparability of the magnitude of the effects.

To make the relative distribution and its decomposition more objective one can calculate entropy based quantification measures and measures of polarization as discussed in Handcock and Morris (1999). Entropy measures can be used to answer the questions “How much does the treatment distribution differ from control distribution?” “How much does the location shift (if any) in the treatment distribution (relative to the control) contribute to the overall difference between the two distributions”.

There are three entropy measures that can be computed

- $D(F_0, F_1)$ the overall entropy i.e. the overall divergence between the treatment and the control distribution
• $D(F_{0L}, F_0)$ the divergence between the location adjusted control distribution and the original control distribution, this measures the effect of location shift

• $D(F_1, F_{0L})$ the divergence between the treatment distribution and the location adjusted control distribution, this measures shape effect.

Note that

$$D(F_0, F_1) \neq D(F_{0L}, F_0) + D(F_1, F_{0L})$$

because of the rescaling needed to compute $D(F_1, F_{0L})$. All entropy measures are based on the Kullback-Leibler divergence measure:

$$D(F_0, F_1) = \int_{-\infty}^{\infty} \log\left(\frac{f_1(y)}{f_0(y)}\right) dF(y) = \int_{0}^{1} \log(g(r)) g(r) dr$$

While the measures just discussed are useful in quantifying how much location and shape differences contribute to the overall difference, to better track shape differences one can use a polarization measure. Following Handcock and Morris (1999) and Contoyannis and Wildman (2007) we use the median-relative polarization (MRP) index. This measure is location (median) adjusted and is used to capture relative polarization. This is especially important in our context because it shows whether some treated students are actually drifting towards the bottom of the achievement distribution and how strong this effect is especially relative to those whose achievement move in the opposite direction. Having accounted for the location effect this measure gives an idea of the relative magnitude of improvement and retardation in achievement.

MPR is given by:

$$MRP(F_0, F_1) = 4 \int_{0}^{1} \left| r - \frac{1}{2} \right| g_{0L}(r) dr - 1$$

This shows that values further away from the centre are given a greater weight with weights increasing linearly with distance from the centre. The measure is re-scaled so it ranges between 1 and −1. Negative values represent less polarization while positive values represent increasing polarization. The index can be interpreted in terms of a proportional shift of mass in the outcome distribution from central to less central values. Increased
polarization in our context means while some learners are gaining other learners are being left behind because it implies that the tails of the outcome distribution are getting fatter.

The polarization index can also be decomposed, this decomposition shows if the polarization represents movement towards the lower or upper tails. The lower and upper tail polarization indices (LPR and UPR, respectively) are given by

\[
LPR(F_0, F_1) = 8 \int_0^{1/2} |r - \frac{1}{2}| g_{0L}(r)dr - 1
\]

\[
UPR(F_0, F_1) = 8 \int_{1/2}^1 |r - \frac{1}{2}| g_{0L}(r)dr - 1
\]

Standard errors can also be calculated for these measures as presented by Handcock and Morris (1999). It is, therefore, possible to test the hypothesis that the polarization is equal to zero.

Results

We start by reproducing the quantile results presented in Fleisch et al, (2017). The authors presented conditional quantile result of the effect of RCU on the total score, spelling score and language score. Their result shows that the programme has no effect on the total score at the mean but has a significant positive effect on the higher quantiles of the spelling and the language scores. While the conditional quantile result suggests the Mathew effect only for the language and spelling scores our unconditional quantile result suggest that there is also evidence of Matthew effect for the total score (see Figure 1).
Table 1: Average Treatment effect Using RIF Regression

<table>
<thead>
<tr>
<th></th>
<th>Overall</th>
<th>Spelling</th>
<th>Language</th>
<th>Comprehension</th>
<th>Writing</th>
</tr>
</thead>
<tbody>
<tr>
<td>TREATMENT</td>
<td>0.65</td>
<td>1.64***</td>
<td>3.82***</td>
<td>-1.78**</td>
<td>0.47</td>
</tr>
<tr>
<td>STANDARD ERROR</td>
<td>0.43</td>
<td>0.51</td>
<td>0.64</td>
<td>0.82</td>
<td>0.75</td>
</tr>
<tr>
<td>EFFECT IN SD</td>
<td>0.032</td>
<td>0.08</td>
<td>0.16</td>
<td>-0.05</td>
<td>0.03</td>
</tr>
<tr>
<td>OBSERVATIONS</td>
<td>2466</td>
<td>2466</td>
<td>2466</td>
<td>2466</td>
<td>2466</td>
</tr>
<tr>
<td>R-SQUARED</td>
<td>0.77</td>
<td>0.7</td>
<td>0.53</td>
<td>0.56</td>
<td>0.37</td>
</tr>
</tbody>
</table>

*P < 0.1, **P < 0.05, ***P < 0.01.

Table 2: Variance treatment effect using RIF regression

<table>
<thead>
<tr>
<th></th>
<th>Overall</th>
<th>Spelling</th>
<th>Language</th>
<th>Comprehension</th>
<th>Writing</th>
</tr>
</thead>
<tbody>
<tr>
<td>TREATMENT</td>
<td>68.06</td>
<td>70.8***</td>
<td>143.6***</td>
<td>20.7</td>
<td>-4.36</td>
</tr>
<tr>
<td>STANDARD ERROR</td>
<td>0.43</td>
<td>24.7</td>
<td>27.6</td>
<td>29.5</td>
<td>25.88</td>
</tr>
<tr>
<td>EFFECT TRADE-OFF</td>
<td>12.69</td>
<td>5.13</td>
<td>3.14</td>
<td>-2.56</td>
<td>-4.44</td>
</tr>
<tr>
<td>OBSERVATIONS</td>
<td>2466</td>
<td>2466</td>
<td>2466</td>
<td>2466</td>
<td>2466</td>
</tr>
<tr>
<td>R-SQUARED</td>
<td>0.53</td>
<td>0.47</td>
<td>0.33</td>
<td>0.35</td>
<td>0.25</td>
</tr>
</tbody>
</table>

*P < 0.1, **P < 0.05, ***P < 0.01.

Negative figures for comprehension and writing indicate that the treatment effect is negative and reduces inequality as measured by variance respectively.

Effect trade-off represents how much variance is incurred for each unit rise/fall in returns.

This highlight efficiency gains when the unconditional quantile is used to estimate the effect. Furthermore, the effect under unconditional quantile method does not vary depending on the covariates. The unconditional quantile results also suggests that there is evidence that only learners at the top quantiles benefit significantly from the programme in every subject and in the overall marks.
The patterns seen in figure 1 are also possible if there is a Matthew effect in the control group too but with less intensity. This is important if this is indeed the case because it means that the normal curriculum is also leaving learners behind.

Table 1 replicates the findings in Fleisch et al. (2017) using RIF regression to estimate the average effect. The results are qualitatively similar to the authors’ results, effect in SD represent is the estimated treatment effect as a function of the standard deviation of the end line scores in the control group.

Table 2 presents the effect of treatment on the variance (a measure of inequality). Note that apart from comprehension and writing score treatment increases variance in the treatment group significantly relative to the control group. This supports the existence of the Matthew effect in the treatment group. Since variance measure departure from the mean, this results suggests that the treatment has some polarizing effect. More importantly, the result in table 2 shows the effect trade-off which captures the amount of variance incurred for each unit increase (or decrease) in returns (EFFECT TRADE-OFF). This estimate is given by the square root of the variance effect estimate divided by the mean impact estimate. For example, the result for the overall score shows that for each 1% increase in mean the variance is increased by 12%. This gives a sense of how strong the polarizing effect is.

Table 3: Treatment effect on the Gini Coefficient

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>TREATMENT</td>
<td>Overall</td>
<td>Spelling</td>
<td>Language</td>
<td>Comprehension</td>
<td>Writing</td>
</tr>
<tr>
<td></td>
<td>0.02**</td>
<td>0.014*</td>
<td>0.02*</td>
<td>0.014*</td>
<td>-0.006</td>
</tr>
<tr>
<td>STANDARD ERROR</td>
<td>0.008</td>
<td>0.009</td>
<td>0.01</td>
<td>0.008</td>
<td>0.006</td>
</tr>
<tr>
<td>OBSERVATIONS</td>
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<td>2466</td>
<td>2466</td>
<td>2466</td>
<td>2466</td>
</tr>
<tr>
<td>R-SQUARED</td>
<td>0.11</td>
<td>0.25</td>
<td>0.31</td>
<td>0.47</td>
<td>0.72</td>
</tr>
</tbody>
</table>

*P < 0.1, **P < 0.05, ***P < 0.01.
Table 3 presents the result for the Gini index. The results agree with table 2 (qualitatively) as the treatment increases inequality in the overall, spelling, language and comprehension scores. To better understand the nature of the inequality in relation to Matthew’s effect we make use of the Generalized Entropy (GE) class of measures. A key feature of this measure is that one can choose the parameter $\alpha$ that assigns weights to distances between outcomes in different parts of the distribution. For lower values of $\alpha$, the measure is more sensitive to changes in the lower tail of the distribution, for higher values, it is more sensitive to changes in the upper tail. The common values of $\alpha$ are 0, 1 and 2 which roughly correspond to the lower, middle and upper part of the distribution.

The result from the class of GE measures is shown in tables 4, 5 and 6. The results show that when $\alpha$ is 0 or 2 the treatment effect has (mostly) significant effect on inequality while when it is 1 effect of treatment on inequality is (mostly) not significant. This suggests that differences in the treatment and control outcome distributions may be more pronounced at the tails. This pattern of results like the result in table 2 (for variance) suggests that the treatment impacts the tails of the outcome distribution.

Table 4: Treatment effect of Generalized entropy measure with (index 0)

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>TREATMENT</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Overall</td>
<td>0.045*</td>
<td>0.04***</td>
<td>0.08***</td>
<td>0.02</td>
<td>-0.03</td>
</tr>
<tr>
<td>Spelling</td>
<td>0.025</td>
<td>0.015</td>
<td>0.02</td>
<td>0.02</td>
<td>0.05</td>
</tr>
<tr>
<td>Language</td>
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<td>2466</td>
<td>1769</td>
<td>2096</td>
<td>1032</td>
</tr>
<tr>
<td>Comprehension</td>
<td>0.06</td>
<td>0.73</td>
<td>0.33</td>
<td>0.06</td>
<td>0.17</td>
</tr>
</tbody>
</table>

*R < 0.1, **P < 0.05, ***P < 0.01.
Relative Distribution Analysis

Results in the last section suggests that there are a lot of differences in the tails of the outcome distribution across treatment arms. To get a better sense of what is going on in the distributions we use the relative density method.

We start our relative distribution analysis by comparing the change within groups. We compare the outcome distribution (for the overall score) in the treatment group before and after treatment using the before distribution as the reference distribution. The motivation for this is that apart from the treatment group there may be evidence of Mathew’s effect in the control group (also). If this is the case then the regular curriculum (control) also leaves some
learners behind. This in part will explain why there is no significant effect at the mean when the treatment group is compared with the control group.

Specifically, let $f_{0B}(y_r)$ be the distribution of outcome in the control group before treatment and let $f_{0A}(y_r)$ be the distribution of outcome in the control group after treatment. $f_{1B}(y_r)$ and $f_{1A}(y_r)$ are similarly defined for the treatment group.

*Figure 2: Relative density for both treatment arms comparing the outcome distributions before and after treatment*

![Relative Density Plot]

*Recall that control group undergo the normal curriculum while the treatment group use the RCUP curriculum. The point here is that one should expect some improvement in the control group too due to the regular curriculum.*
be uniformly distributed, the height of the decile plots will be 1 and the entropy value will be zero.

The relative distribution in figure 2 suggests that there has been a shift to the right in both distributions with this shift being significant at the upper tails. To see this note that the relative density is greater than one in the upper quantiles which means that in the ratio $\frac{f_{0A}(y_{0A})}{f_{0B}(y_{0B})}$ and $\frac{f_{1A}(y_{1A})}{f_{1B}(y_{1B})}$, the numerator has larger density than the denominator in both groups. Specifically, the mass in the 10th decile after treatment is about twice the mass before treatment. This in summary means that there is improvement in marks for some learners in both the treatment and the control group. This should be expected since both groups undergo training howbeit under different curricula.

Figure 3: Within-group decomposition (treatment group) into mean and shape effects

The result in figure 2 shows the total difference between the two distributions (i.e. without decomposition). Figures 3 & 4 show the decomposition of the change in figure 2 into the location (mean\textsuperscript{7}) effect and the shape effect. The decomposition can be written as

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\textsuperscript{7}We use the mean instead of the median because we are using randomized data and interest is often on the average treatment effect.
\[
\frac{f_{1A}(y_\tau)}{f_{1B}(y_\tau)} = \frac{f_{1M}(y_\tau)}{f_{1B}(y_\tau)} \times \frac{f_{1A}(y_\tau)}{f_{1M}(y_\tau)}
\]

Where \( f_{1M}(y_\tau) \) is the distribution one gets when the mean of the treatment distribution before treatment is adjusted to coincide with the mean of the treatment distribution after treatment. The first term is the location effect while the second term is the shape effect.

The second panel in figure 3 shows that there has been a positive mean shift in the treatment group, suggesting that there has been a positive improvement for this group. The third panel of figure 3 show that net of the mean shift, there has been significant difference in the shape of the treatment distribution. The pattern of the shape difference suggests the existence of polarization (or Matthew’s effect in this case) i.e. there are more observation at the tails when the treatment group is compared before and after treatment (note that the polarization is particularly strong at the lowest decile).

The entropy measure in figure 3 clearly shows that the change in the outcome distribution for treated learners is driven by the shape component of the decomposition (0.12 versus 0.41). This suggests that net of the increase in mean, the difference in the two distributions is driven by change is shape (in the form of polarization) of the distribution of outcome. This quantitatively supports the existence of Matthew effect (note that we have come to this conclusion without comparing with the control outcomes).

Since the location effect is small relative to the shape effect one can conclude that the change induced by both the treatment (RCUP programme) and time is such that inequality in achievement has increased (providing some context to results in tables 2 to 6). The polarization indices discussed in the method section can be used to help quantify the shape component of the decomposition. Therefore, the MPR, LPR and UPR values for panel 3 of figure 3 are shown in the first row of table 7. The polarization indices are positive indicating movement of mass in the distribution away from the centre (polarization). Polarization is stronger in the lower tail (0.49) relative to the upper tail (0.14) as suggested by figure 3.

**Table 7: MRP and its decomposition (Overall outcome)**

<table>
<thead>
<tr>
<th></th>
<th>LRP</th>
<th>MRP</th>
<th>URP</th>
</tr>
</thead>
</table>

\(^8\) Note that \( \frac{f_{1A}(y_\tau)}{f_{1B}(y_\tau)} = \frac{f_{1A}(y_{1A})}{f_{1B}(y_{1B})} \)
<table>
<thead>
<tr>
<th></th>
<th>LB</th>
<th>Est</th>
<th>UP</th>
<th>LB</th>
<th>Est</th>
<th>UP</th>
<th>LB</th>
<th>Est</th>
<th>UP</th>
</tr>
</thead>
<tbody>
<tr>
<td>treatment(before &amp; AFTER)</td>
<td>0.40</td>
<td>0.49</td>
<td>0.58</td>
<td>0.26</td>
<td>0.31</td>
<td>0.36</td>
<td>0.04</td>
<td>0.14</td>
<td>0.23</td>
</tr>
<tr>
<td>Control(before &amp; AFTER)</td>
<td>0.35</td>
<td>0.42</td>
<td>0.50</td>
<td>0.24</td>
<td>0.29</td>
<td>0.33</td>
<td>0.06</td>
<td>0.15</td>
<td>0.23</td>
</tr>
<tr>
<td>Treatment vs Control</td>
<td>0.04</td>
<td>0.12</td>
<td>0.22</td>
<td>0.04</td>
<td>0.08</td>
<td>0.13</td>
<td>-0.04</td>
<td>0.04</td>
<td>0.14</td>
</tr>
</tbody>
</table>

The conclusion that there is evidence of Matthew’s effect in form of polarization of the outcome distribution is identical to the conclusion in Fleisch et al, (2017). However, the authors came to the conclusion by comparing the performance of treated learners with control learners. Our result shows that the same conclusion can be reached by comparing outcome distribution within the treatment group (i.e. before and after treatment). Another important point to note is that the conclusion in Fleisch et al, (2017) suggests that it is the RCUP programme that induced the heterogeneous impact (Matthew effect) and is silent on how the regular curriculum (used for the control group members fare in terms of Matthew effect). The result of the within-group decomposition of the control group shed some light on this question.

Figure 4 shows the within-group decomposition for the control group. The decomposition results are similar to the one in figure 3 i.e. net of the mean effect, there is evidence of polarization in the control group relative to the baseline outcome. The entropy measure suggests that a larger portion of the change in outcome distribution is driven by the shape component decomposition. The polarization indices in the second row of table 7 also suggests that polarization is stronger at the lower tails.

These results provides a plausible explanation for why the RCUP programme has no significant/meaningful effect on the mean outcome. First, the change in the distribution of outcome in both groups is mostly driven by the shape component. Second, the change in the distribution of outcome relative to the baseline is very similar for both groups. This point to the fact that both the RCUP and the regular curriculum induce Matthew’s effect on the outcome. This is important because ideally, a social planner may be interested in improving the performance of learners at the lower deciles of the outcome distribution. The existence of the Matthew effect in both treatment arms does not improve the utility of such a social
planner. Specifically, RCUP is different from the regular curriculum in that it is a remedial programme designed to help students catch up. This suggests that the way it should impact the outcome distribution should be different from the regular curriculum. Evidence that its effect is similar to the regular one suggests that the cost associated with it may not be justified.

There is however an important difference in the effect of both curricula on the outcome distributions. Comparing row 1 (decomposition in the treatment group) of table 7 with row 2 (decomposition in the control group) suggests that polarization is stronger in the treatment group (0.31 versus 0.29), specifically at the lower quantile (0.49 versus 0.42). However, at the upper decile polarization seem to be slightly stronger in the control group (0.14 versus 0.15).

**Figure 4:** Within-group decomposition (control group) into mean and shape effects

Lastly, we compare the distribution of outcome in the treatment and control groups after treatment. Given the results in figures, 3 and 4 one should not expect much difference between the outcome distributions across treatment arms after treatment. The result is shown in figure 5. Note that in this case we are comparing the outcome of the control and the treatment group after treatment. Therefore the decomposition can be written as
where \( f_{1A}(y_r) \) and \( f_{0A}(y_r) \) is the distribution of outcome in the treatment and control groups after treatment and \( f_{0M}(y_r) \) is the counterfactual distribution obtained when the (mean) treatment effect is added to the control distribution \( f_{0A}(y_r) \).

\[
\frac{f_{1A}(y_r)}{f_{0A}(y_r)} = \frac{f_{0M}(y_r)}{f_{0A}(y_r)} \times \frac{f_{1A}(y_r)}{f_{1B}(y_r)}
\]

Figure 5: Across groups decomposition (control versus treatment group) into mean and shape effects.

The total difference (first panel of figure 5) suggests that there is more mass in the 10th decile of the treatment distribution relative to the control. The second panel suggests that there is hardly any mean change while the third panel show that the treatment distribution is more polarized than the control distribution after treatment. This is important because it suggests that while there is polarization in both groups (figures 3 and 4), polarization is stronger in the treatment group. The shape component is figure five suggests that more learners benefit from RCUP relative to the control group (upper tail polarization) but at the same time more

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9 Note that \( \frac{f_{1A}(y_r)}{f_{1B}(y_r)} \frac{f_{1A}(y_{1A})}{f_{1B}(y_{1B})} \)
learners are being left behind in the treatment group relative to the control. In other words the tails of the treatment distribution after treatment is heavier than the tails of the control distribution.

Row 3 of table 7 shows the polarization results. The overall polarization (MPR) and the lower tail polarization (LRP) are both positive and significant, while the upper tail polarization (URP) is not significant. This suggests that movement towards the lower tail in the treatment distribution is an important driving force when it comes to the difference between the treatment and the control after treatment. The implication of this results in that the Mathew's is stronger in the treatment group suggesting that more students are being left behind in this group. Note that this results agree with the RIF results in tables 2 to 6. The RCUP programme tend to increase inequality in the distribution of outcome for beneficiaries relative to the control.

Our analysis so far is based on the overall performance of the learners i.e. weighted average of the marks from spelling, language, comprehension and writing. At the mean the RCUP programme has no significant effect on the overall scores and our relative density analysis suggest that this is because of polarization in both groups. However the dynamics might be different for different subjects (disaggregated marks). For example at the mean RCUP has the highest (positive) significant effect on language while its effect on writing is positive but insignificant (see table 1). To see is there are differences in polarization effect in disaggregated marks we perform similar analysis for language and writing to see how these treatments affect the outcome differently across the treatment arms.

It is important to note that the overall score was modelled as a continuous distribution because it represents weighted averages of discrete marks. The marks for the different subjects are better modelled as discrete variables (the marks assume values of 10, 20,….100). Therefore the language and writing marks are modelled as discrete variables. Handcock and Morris (2006, chapter 11) discuss how to implement the methods discussed in section (Methods) for discrete or grouped data.

We start with spelling scores, figure 6 shows the before and after decomposition for the treatment spelling scores
Unlike figure 3 (treatment distribution for the overall score) the mean effect is stronger than the shape effect for the spelling score. This perhaps explains why there is significant mean difference for this outcome. Furthermore, recall that in figure 3 the LRP (lower tail polarization) is stronger than the URP (upper tail polarization). In figure 6 the effect is reversed with polarization being stronger at the upper tail (see table 8).

Table 8: MRP and its decomposition (Overall outcome)

<table>
<thead>
<tr>
<th></th>
<th>LRP</th>
<th>MRP</th>
<th>URP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LB</td>
<td>Est</td>
<td>UP</td>
</tr>
<tr>
<td>treatment (before &amp; AFTER)</td>
<td>N/A</td>
<td>0.13</td>
<td>N/A</td>
</tr>
<tr>
<td>Control (before &amp; AFTER)</td>
<td>N/A</td>
<td>-0.09</td>
<td>N/A</td>
</tr>
<tr>
<td>Treatment vs Control</td>
<td>N/A</td>
<td>0.004</td>
<td>N/A</td>
</tr>
</tbody>
</table>

*Note that the r package “reldist” does not compute lower and upper bound for LRP and UPR
Figure 7 present the result for the control distribution of the spelling scores. Similar to figure 4 (control distribution for the overall score) the shape component is the most important. Unlike figure 4 polarization seem to be stronger at the upper tail. It appears that the major difference is that polarization in the language scores is one sided (upper tail polarization in both groups). This polarization effect coupled with a stronger mean component in the treatment group tend to be the driving force behind the positive mean difference for the spelling scores.

The net effect is shown in figure 8. The difference in the treatment and control distributions is largely driven by the shape component and this shape component is one sided i.e. upper tail polarization (also see table 8 row 3).
For the writing scores we only present the result for the comparison between the treatment and control outcome (after treatment\textsuperscript{10}). The net effect explains why there is no significant effect. The total difference is very small compared to other results, the first panel of figure 9 shows that the relative density comparing the spelling scores across groups is uniformly distributed. Both the mean and shape effects are very small, and the shape effect suggests that there is no significant polarization effect in the treatment group relative to the control group.

These results suggests that the major difference between the overall scores and the spelling and writing scores is the nature of the polarization effects. While the location effect can explain some portion of the total change in distribution the major differences are driven by the polarization effect. Where there are no significant mean effect the polarization is in both directions suggesting while some learners benefit from the programme others are being left behind. Particularly our within group comparison show that the lower tail polarization is

\textsuperscript{10} The result for the within group comparisons is not very different from other results presented. Both the treatment and control group have positive polarization at the upper tails.
stronger than the upper tail polarization for the overall scores. The across group comparison suggests that polarization is stronger in the treatment group relative to the control. In the case where there is no significant mean difference (writing scores), positive polarization is both groups tend to cancel out in the final comparison.

*Figure 9: Writing: Across groups decomposition (control versus treatment group)*

**Conclusion**

Simply looking at the mean effect of an intervention is likely to hide crucial nuances. Following Deaton and Cartwright’s critique, we investigate the impact of an education intervention on the distribution of learning outcomes among primary school learners. Our results show that there is polarization in both groups (not only treatment group). Thus, while there is a natural increase in learning inequality over time in South African primary schools which might be due to the misalignment of curriculum and learner profile, the instructional change intervention has exacerbated the polarization and increased inequality in learning outcomes. This is highly problematic as educational outcomes already determine
life and work trajectories in the South African society. Increasing learning inequality through policies that advance higher performing learners at the cost of low performing learners will only increase the social ills of South Africa. The implications of these findings might point at yet another issue in line with the argument by Kaffenberger and Pritchett (2019): the South African curriculum is misaligned with the learner profiles. Thus, while the instructional change intervention was supposed to test a possible solution to the education problem by improving the instructional practice of primary school teachers, the findings are more likely to have unearthed other problems around teaching at the right level. Issues that need further investigation.
References


Contoyannis and Wildman (2007)


Firpo (2005)

Fleisch et al. (2017)

Fortin and Lemieux (2009)

Handcock and Morris (1999)

Heckman et al. (1997)
