On Local Automorphisms of SL_n

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Abstract: A linear map Δ is said to be a local automorphism if for all x there exists some automorphism $\phi(x)$, such that $\Delta(x) = \phi_x(x)$. It has been proven for an associative algebra of square matrices $M_n(\mathbb{R})$ that any local automorphism is either an automorphism or an anti-automorphism [1]. Furthermore, Ayupov et. al. have classified local automorphisms of simple Leibniz algebras proving that if Δ is a local automorphism over a complex simple Leibniz algebra $L = G + I$, then Δ is an automorphism if and only if its Lie part $\Delta_{G,G}$ is an automorphism [2]. This result simplifies the classification of local automorphisms of simple Leibniz algebras down to the Lie case and gives us reason to classify local automorphisms of Lie algebras. In our work we consider the simple Lie algebra of traceless n x n matrices (sl_n). There is a well-known classification of automorphisms of simple Lie algebras which states that over an algebraically closed field of characteristic 0, the group of automorphisms of the Lie algebra sl_2 is the set of mappings $X \to A^-1XA$ and the group of automorphisms of the Lie algebra sl_n ($n \geq 3$) is the set of mappings $X \to A^{-1}XA$ and $X \to -AX^\top A^{-1}$ [3]. Using this fact, our motivation is to classify local automorphisms of the simple Lie algebra sl_n and see if it is possible to obtain results similar to those for associative algebras.

For the Lie algebra of 2 x 2 matrices with trace zero, sl_2, we are able to classify all local automorphisms that fix the basis element $h = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ and we obtain the following result:

Theorem: The only local automorphisms of sl_2 are all automorphisms and all anti-automorphism.

We also construct a local automorphism for sl_n ($n \geq 3$) that is neither an automorphism nor an anti-automorphism and prove the following collorary:

Collorary: For $n \geq 3$ we prove that AUT$\pm(sl_n)$ is a proper subgroup of LAut(sl_n) of an infinite index.

We do not have a full description of the group LAut(sl_n), but we find that obtaining a full description is a much bigger problem that may or may not be possible of obtaining. Thus, we find different results from those for associative algebras. If we do obtain a full description, we can find if this group is algebraic or not.

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